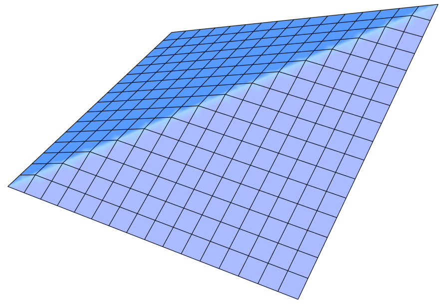

ACTSC 371: Corporate Finance I

Prof. Brent Matheson • Winter 2014 • University of Waterloo
MWF 9:30–10:20AM in DC 1351

Transcribed, LaTeXed, and edited by Rob Zimmerman

Note: These are personal lecture notes, not official course materials. They may contain errors (which by default you should attribute to me, Rob Zimmerman, rather than the course instructor). The permanent home of these — and all of my other lecture notes — is `https://rob-zimmerman.github.io/coursenotes`.

Contents

Lecture 1 — Monday, January 06, 2014

1. Corporate Finance Foundations

Introduction to Corporate Finance

Corporate finance revolves around three large questions. In which long-lived assets should the firm invest? How should it raise the cash needed for those investments? And how should it manage the short-term operating cash flows that keep the business running? The third question is essentially a net working capital problem.

The financial manager's central job is to create value: choose worthwhile investments, finance them sensibly, and make sure the operating side has enough short-term liquidity to function. The formal valuation tools come later, but the managerial perspective is already visible—we want to organize and finance the firm in a way that increases its total value.

Positive net present value is the primary benchmark for decision making. Many alternative decision rules appear later, but net present value remains the reference criterion because it asks directly whether a project increases the value of the firm.

The **balance sheet model of the firm** organizes these decisions. The asset side contains current and fixed assets, with the fixed assets divided into tangible and intangible components; the financing side divides the same total firm value among current liabilities, long-term debt, and shareholders' equity.

Category	Asset Side	Financing Side
Overall view	Total value of assets	Total firm value to investors
Short-term claims	Current assets	Current liabilities
Long-term claims	Fixed assets	Long-term debt
Residual detail	Tangible and intangible asset components	Shareholders' equity

The balance sheet model encodes three major decisions:

1. The **capital budgeting decision** determines which long-term assets the firm should acquire.
2. The **capital structure decision** determines how the firm should finance those assets.
3. The **net working capital decision** determines how much short-term operating liquidity the firm needs to pay its bills.

The first question is about the left-hand side of the balance sheet, because it asks which real assets the firm should own. The second question is about the right-hand side, because it asks how those assets should be financed. The third question sits in between operations and financing, because a firm can be profitable in the long run and still encounter trouble if it cannot meet short-term obligations as they come due.

Remark 1.1 Firm value can be viewed as a pie. The manager's goal is to increase the size of the pie, while capital structure asks how to slice it between debt and equity. If the way the pie is sliced affects the total size of the pie, then the capital structure decision matters for value as well.

The Firm and Its Cash Flows

In a large corporation, the financial function is usually coordinated through the **vice president of finance** or **chief financial officer**. Two standard branches below that officer are the **treasurer** and the **controller**. The treasurer is typically associated with cash management, credit management, capital expenditures, and financial planning, while the controller is typically associated with financial accounting, cost accounting, tax management, and data processing. The organizational chart itself is less important than the idea that the finance function touches both forward-looking decisions and record keeping.

The firm also sits inside the broader cash flow system in Figure 1. It raises funds in financial markets, invests them in assets, generates operating cash flows, pays taxes, and distributes cash to investors through dividends and debt payments; whatever it retains can be reinvested.

Ultimately, the firm must generate cash. Cash raised from investors has to be invested productively enough that the resulting cash flow can cover payments to creditors, payments to shareholders, taxes, and any additional reinvestment the firm chooses to undertake.

Corporate securities can be treated as contingent claims on total firm value. If the firm promises debt holders a fixed amount F and the total firm value at the relevant date is X, then debt and

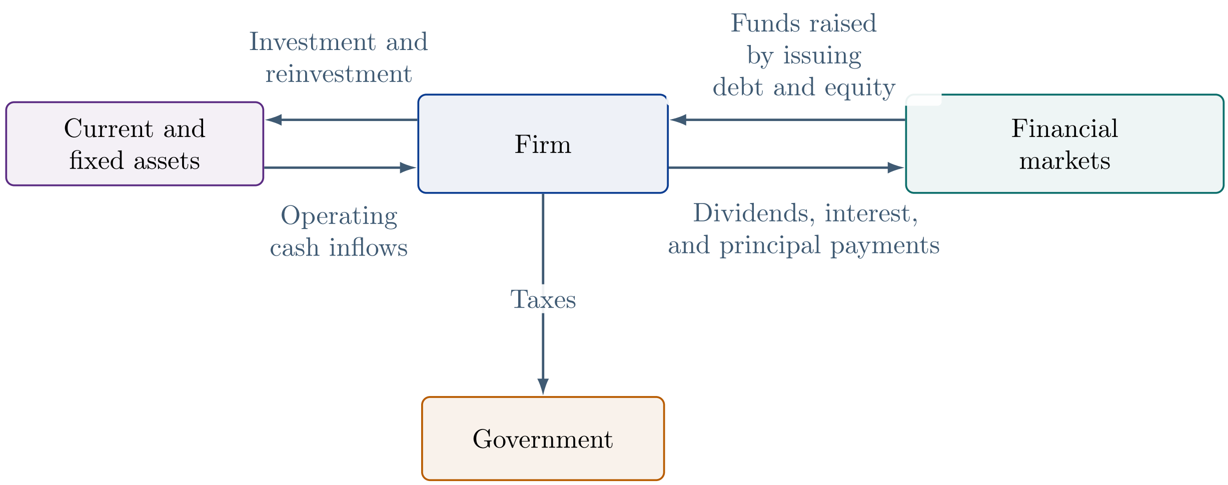

FIGURE 1

equity have the complementary payoff profiles shown in Figure 2. The debt holders' claim is

$$\min(F, X),$$

while the shareholders' claim is

$$\max(0, X - F).$$

Thus the shareholders are residual claimants. If firm value is less than the promised debt payment, shareholders get nothing. If firm value exceeds the promised debt payment, shareholders get everything above F.

FIGURE 2

The two claims fit together exactly. When the firm is worth less than F, debt holders absorb the entire firm value and equity is worthless. When the firm is worth more than F, debt holders stop at F and equity takes the upside. In Figure 3, the two coloured segments therefore join into the 45° line representing total firm value.

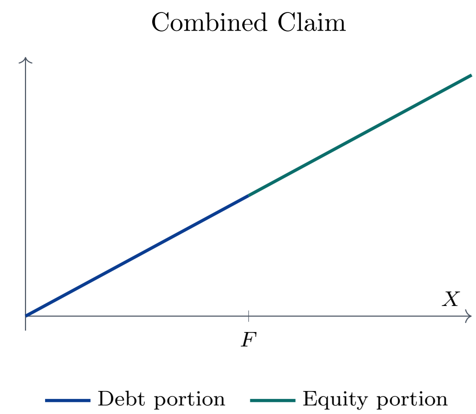

Figure 3

Algebraically, this decomposition is

$$\min(F, X) + \max(0, X - F) = X.$$

It is worth remembering this identity because it foreshadows later option-based interpretations of corporate securities.

Business Organization and Agency

The corporate form is the standard way to raise large amounts of cash, but it is not the only form of business organization. The main forms are sole proprietorships, partnerships, corporations, and income trusts. The comparison matters because the legal form affects control, liability, taxation, and the ease with which ownership can be transferred.

In a **sole proprietorship**, ownership and control are concentrated in one person, which makes decision making simple but severely limits access to capital and does not provide the same liability protection as a corporation. **Partnerships** broaden the ownership base, but they still face restrictions, especially in general partnerships where the general partners may have unlimited liability. A corporation, by contrast, allows ownership to be split into shares that can be traded much more easily.

Criterion	Corporation	Partnership
Liquidity and marketability	Shares can usually be exchanged relatively easily.	Ownership transfers are subject to substantial restrictions.
Control	Voting rights are usually attached to shares.	The general partner is in charge, while limited partners have much less control.
Liability	Shareholders have limited liability.	General partners may have unlimited liability, although limited partners have limited liability.
Continuity of existence	The corporation has perpetual life.	The partnership has limited life.
Reinvestment policy	The firm has broad latitude in retaining cash.	Net cash flow is often distributed more directly to partners.
Taxation	Corporate income is taxed at the corporate level, with dividend tax treatment applied later.	Partnership income is generally passed through to partners for taxation.

These characteristics explain why the corporate form dominates finance on a large scale. Liquidity, continuity, and limited liability all make it easier to attract outside investors.

Corporate finance traditionally treats the maximization of shareholder wealth as the firm's organizing objective. This does not mean mechanically maximizing accounting profit in a single period; it means choosing policies that increase the market value of the shareholders' claim.

We can view the firm as a set of contracts, among which the relationship between shareholders and managers is especially important. Shareholders supply capital and bear residual risk, but they usually do not run the daily business; management makes operating and financing decisions on their behalf.

This creates an agency relationship between shareholders and managers. Managers may not always have the same incentives as shareholders, so agency costs can arise. Standard examples include expensive perks, an excessive focus on managerial survival or independence, and growth for its own sake even when that growth does not increase shareholder wealth.

The separation of ownership and control can be summarized as follows: shareholders elect the board of directors, the board hires and oversees management, management controls the firm's assets, and both debt holders and shareholders have claims on the resulting cash flows. The structure works well when incentives are aligned, but it creates room for conflict whenever managers can pursue private objectives at shareholders' expense.

Several mechanisms are used to limit such **agency problems**:

1. Compensation contracts can be designed to align managerial incentives with shareholder objectives.
2. Shareholders and boards can monitor management directly.
3. The market for managerial talent can discipline underperforming managers.
4. The market for corporate control can discipline management through the threat of takeover.

Agency costs may be direct, such as monitoring and contracting costs, or indirect, such as value lost when management makes suboptimal decisions. Either way, they matter because any increase in agency costs reduces firm value. Stakeholders other than shareholders and managers, including employees, customers, suppliers, and the public, also have a financial interest in corporate decisions. This raises ethical and socially responsible investing questions, even though the formal decision criterion remains shareholder wealth maximization.

Financial Institutions and Markets

Financing can occur either indirectly through financial intermediaries or directly through financial markets. In **indirect finance**, funds move from suppliers of capital into intermediaries such as banks, and then from those intermediaries to firms and households that demand funds. In **direct finance**, suppliers of funds purchase securities issued by firms through markets such as money markets and capital markets.

Money markets are for short-term debt instruments, whereas **capital markets** are for long-term debt and equity. This distinction matters because short-term liquidity management and long-term financing are not the same problem, even though both belong to finance.

Primary and secondary markets should also be kept distinct. In a **primary market**, the corporation issues securities and receives cash from investors, usually with the help of an underwriter. In a **secondary market**, investors trade previously issued securities among themselves. The firm is not directly raising new money in the secondary market, but that market still matters because liquid secondary trading makes it easier for the firm to issue securities in the first place.

Markets can also differ by trading mechanism. Some securities are **exchange-traded**, while others trade **over the counter** through dealer markets. Foreign exchange is an especially important example of an over-the-counter market and is the world's largest financial market. Listing on an organized exchange can improve trading liquidity and make equity financing easier, although firms must satisfy listing requirements and, in some cross-listed settings, substantial disclosure requirements.

Financial markets have become increasingly integrated and global, while financial engineering has been used to reduce costs associated with risk, taxes, and financing. Improved computing technology has also made economies of scale and scope more important. At the same time, deregulation, greater volatility, and the aftereffects of financial crises have made financial management more technical and more complex.

Lecture 5 — Wednesday, January 15, 2014

2. Financial Statements and Planning

Accounting Statements

Accounting statements are foundational because they feed directly into cash flow calculations and valuation. The objective is to extract the financial quantities needed for corporate finance decisions.

The raw material for financial analysis comes from public filings, market data, and commercial databases. Balance sheets, income statements, ratio summaries, and security prices are not abstract objects: they are disclosures that analysts must locate, read, and reconcile. Sources include Statistics Canada, securities regulators, stock exchanges, SEDAR filings, and commercial financial databases.

The balance sheet is an accountant's snapshot of the firm's accounting value at a particular date, and it always satisfies the identity

$$\text{Assets} \equiv \text{Liabilities} + \text{Stockholders' Equity}.$$

Assets are generally listed in order of **liquidity**, that is, by the speed with which they can normally be converted into cash. Thus cash is more liquid than accounts receivable, which is more liquid than inventory, while property, plant, and equipment are much less liquid still.

Example 5.1 Consider a hypothetical manufacturing firm whose compact balance sheet extract is:

	2009	**2008**
Cash and equivalents	140	107
Accounts receivable	294	270
Inventories	269	280
Other current assets	58	50
Total current assets	761	707
Net property, plant, and equipment	873	814
Intangible assets and other	245	221
Total assets	1,879	1,742

	2009	**2008**
Accounts payable	213	197
Notes payable	50	53
Accrued expenses	223	205
Total current liabilities	486	455
Deferred taxes	117	104
Long-term debt	471	458
Total long-term liabilities	588	562
Preferred shares	39	39
Common shares	376	339
Accumulated retained earnings	390	347
Total equity	805	725
Total liabilities and equity	1,879	1,742

This example illustrates two recurring conventions. We read down the asset side from most liquid to least liquid, and we read the liability-and-equity side as an ordering of claims: short-term obligations first, then longer-term debt claims, and finally the shareholders' residual claim.

Three concerns matter when reading a balance sheet:

1. Liquidity.
2. Debt versus equity.
3. Value versus cost.

Accounting liquidity refers to the ease and speed with which assets can be converted to cash. The more liquid a firm's assets are, the less likely the firm is to experience difficulty meeting short-term obligations. At the same time, very liquid assets often earn relatively low returns compared with long-lived operating assets, so liquidity provides safety at the cost of lower expected profitability.

Debt versus equity is about the ordering of claims. When a firm borrows, creditors typically receive the first claim on cash flow and assets. Shareholders are therefore residual claimants, so shareholder equity is the amount left over after liabilities are subtracted from assets.

Value versus cost is the warning that accounting numbers are not market prices. Under IFRS, the accounting value of an asset is usually its carrying value or book value, whereas market value is the price at which willing buyers and sellers would actually trade the asset. The latter is usually the relevant concept for decision making.

Common shares together with retained earnings make up shareholders' equity. If we move from one balance sheet date to the next, then

$$TE_2 = TE_1 + \Delta RE + \Delta \text{Common Shares},$$

where

$$RE_2 = RE_1 + NI - \text{Dividends paid.}$$

Thus additions to retained earnings are net income minus dividends. Figure 4 separates the internal and external sources of the change in total equity.

This sketch separates the two ways shareholders' equity grows over time: internally through retained earnings and externally through new equity financing.

Example 5.2 For this firm, net income in 2009 was 86 and dividends were 43, so additions to retained earnings were also 43. This matches the increase in accumulated retained earnings from 347 to 390. Over the same period, common shares increased from 339 to 376, so total

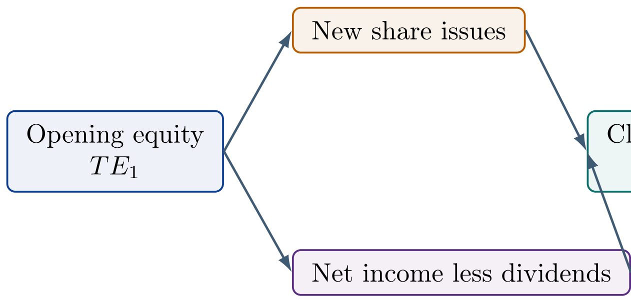

FIGURE 4

equity rose both because profits were retained and because new equity was issued.

Example 5.3 Suppose net income is 45,000 and dividends are 10,000. Then additions to retained earnings are 35,000. If common shares remain 125 and retained earnings rise from 75 to 110, then shareholders' equity rises from 200 to 235.

Remark 5.4 Balance sheet identities provide a quick internal consistency check.

For fixed assets, a presentation on a gross basis tracks fixed assets, accumulated depreciation, and net fixed assets:

$$\text{Net Fixed Assets} = \text{Fixed Assets} - \text{Accumulated Depreciation}.$$

Depreciation reduces accounting income and book value, but it is not itself a cash outflow during the period.

	2007	**2008**
Fixed Assets	25,000	27,500
Accumulated Depreciation	(10,000)	(13,500)
Net Fixed Assets	15,000	14,000

This is one reason net fixed assets can fall even during a year in which the firm invests in new property, plant, and equipment. Depreciation pushes book value down, while capital spending pushes it up. Looking only at the net figure hides the gross flows.

The **income statement** records the firm's performance over a period and satisfies the identity

$$\text{Income} \equiv \text{Revenue} - \text{Expenses}.$$

Example 5.5 The same firm's income statement is:

Item	**2009**
Total operating revenues	2,262
Cost of goods sold	(1,655)
Selling, general, and administrative expenses	(327)
Depreciation	(90)
Operating income	190
Other income	29
Earnings before interest and taxes	219
Interest expense	(49)
Pretax income	170
Taxes: current and deferred	(84)
Net income	86
Retained earnings	43
Dividends	43

The operations section of the income statement reports revenues and expenses from principal operations. The non-operating section contains financing-related items such as interest expense. Taxes are then reported separately, and net income is the bottom line.

Common earnings measures are:

1. **EBITDA**: Earnings before interest, taxes, depreciation, and amortization.
2. **EBIT**: Earnings before interest and taxes (often called **operating profit**).
3. **EAT**: Earnings after tax (that is, **net income**).

The key practical point is that depreciation is a non-cash charge. It reduces taxable income, and therefore taxes, but it is added back when computing operating cash flow.

Three cautions matter when reading an income statement:

1. Under IFRS, revenue and expense recognition follows the **matching principle**, so income is reported when it is earned rather than when the cash is actually received.
2. Non-cash items matter. Depreciation is the clearest example, and deferred taxes are another item that does not correspond one-for-one with current-period cash flow.
3. Time and cost structure matter. In the short run, some inputs are effectively fixed; in the long run, all inputs are variable. Financial accounting does not organize costs in quite the same way as economic cost analysis.

Remark 5.6 Book value and market value are distinct. Under IFRS, the accounting value of an asset is its carrying value or book value, whereas market value is the price at which willing buyers and sellers would actually trade the asset.

Cash Flow from Assets

Net working capital is the difference between current assets and current liabilities:

$$NWC \equiv CA - CL.$$

It is positive when current assets exceed current liabilities. A growing firm often has a positive addition to net working capital because receivables, inventories, and other short-term operating balances tend to expand with sales.

Example 5.7 For this firm,

$$NWC_{2008} = 707 - 455 = 252,$$

and

$$NWC_{2009} = 761 - 486 = 275.$$

Therefore,

$$\Delta NWC = 275 - 252 = 23.$$

This increase of 23 is an investment of cash by the firm, because more funds are tied up in short-term operating balances.

Writing current cash taxes rather than total tax expense, one formula for operating cash flow is

$$OCF = EBIT + \text{Depreciation} - \text{Current Taxes}.$$

An equivalent bottom-up formula adds back interest and all relevant noncash charges:

$$OCF = NI + \text{Interest} + \text{Depreciation} + \text{Other Noncash Charges}.$$

For the statements below, the additional noncash charge is the deferred-tax expense. Omitting it would make the two routes disagree by 13.

Cash flow from assets, also interpreted as free cash flow to the firm, is

$$CF(A) = OCF - \Delta NWC - \text{Net Capital Spending}.$$

The important warning is that net capital spending cannot be inferred from the change in net fixed assets alone, because depreciation also changes net fixed assets.

Remark 5.8 If $\Delta FA = -4000$, it does not necessarily mean that the firm sold 4000 of assets. The firm could instead have recorded 6000 of depreciation and 2000 of new purchases.

A key identity is the "Treasurer's Rule":

$$CF(\text{Assets}) = CF(\text{creditors}) + CF(\text{stockholders}).$$

More specifically,

$$CF(B) = \text{Interest} + \text{Long-term debt redemption} - \text{New debt issued},$$

and

$$CF(S) = \text{Dividends} + \text{Stock repurchases} - \text{Stock issuance}.$$

Example 5.9 The firm's financial cash flow can be summarized as follows:

Cash Flow of the Firm	2009
Operating cash flow	238
Capital spending	(173)
Additions to net working capital	(23)
Cash flow from assets	42
Cash flow to creditors	36
Cash flow to stockholders	6
Total cash flow to investors	42

This table puts the accounting statements into a finance format. We first compute the cash produced by operations, subtract the amounts tied up in capital spending and working capital, and then check that the remainder is exactly the cash paid to creditors and shareholders.

Remark 5.10 A standard workflow starts with an income statement together with two consecutive balance sheets, then reconstructs net income, operating cash flow, changes in working capital, and capital spending.

Example 5.11 We can verify the financial cash flow table directly. Operating cash flow is

$$OCF = EBIT + \text{Depreciation} - \text{Current Taxes} = 219 + 90 - 71 = 238.$$

Capital spending is

$$\text{Purchase of fixed assets} - \text{Sales of fixed assets} = 198 - 25 = 173.$$

If net working capital rises from 252 to 275, then additions to net working capital equal 23. Hence

$$CF(A) = 238 - 173 - 23 = 42.$$

Cash flow to creditors is

$$CF(B) = 49 + 73 - 86 = 36,$$

while cash flow to stockholders is

$$CF(S) = 43 + 6 - 43 = 6.$$

Therefore,

$$CF(A) = CF(B) + CF(S) = 36 + 6 = 42.$$

Example 5.12 A recurring identity is

$$\Delta TE = NI - \text{Dividends} + \text{New Shares}.$$

If total equity rises from 250,000 to 350,000, then $\Delta TE = 100{,}000$. If dividends are 15,000 and new shares issued are 40,000, then

$$100{,}000 = NI - 15{,}000 + 40{,}000,$$

so $NI = 75{,}000$.

Financial Ratios

Ratios focus attention on relationships that may remain informative even when raw financial statement amounts change significantly from year to year. The main categories are short-term solvency, activity, financial leverage, profitability, and value. We generally use average values in denominators when a ratio describes performance over a period rather than at a single date.

We begin with short-term liquidity measures.

Definition 5.13 The **current ratio** is

$$\text{Current Ratio} = \frac{\text{Current Assets}}{\text{Current Liabilities}}.$$

The **quick ratio** is

$$\text{Quick Ratio} = \frac{\text{Quick Assets}}{\text{Current Liabilities}},$$

where **quick assets** are current assets minus inventories. The **cash ratio** is

$$\text{Cash Ratio} = \frac{\text{Cash and Equivalents}}{\text{Current Liabilities}}.$$

The quick ratio is meant to measure the firm's ability to cover current liabilities without relying on the sale of inventory.

Activity and profitability ratios connect revenue, assets, and income.

Definition 5.14 The **total asset turnover ratio** is

$$\text{Total Asset Turnover} = \frac{\text{Total Operating Revenue}}{\text{Average Total Assets}}.$$

The **receivables turnover ratio** is

$$\text{Receivables Turnover} = \frac{\text{Total Operating Revenue}}{\text{Average Receivables}}.$$

The **average collection period** is

$$\text{Average Collection Period} = \frac{\text{Days in Period}}{\text{Receivables Turnover}}.$$

The **inventory turnover ratio** is

$$\text{Inventory Turnover} = \frac{\text{Cost of Goods Sold}}{\text{Average Inventory}}.$$

The **days sales in inventory** measure is

$$\text{Days Sales in Inventory} = \frac{\text{Days in Period}}{\text{Inventory Turnover}}.$$

The **net profit margin** is

$$\text{Net Profit Margin} = \frac{\text{Net Income}}{\text{Total Operating Revenue}}.$$

A higher current ratio indicates greater short-term liquidity, while a longer collection period signals a larger average investment in accounts receivable. Retail and wholesale firms often have relatively high asset turnover, whereas service firms often have higher profit margins.

For longer-term solvency analysis, we turn to leverage ratios.

Definition 5.15 The **debt-equity ratio** is

$$\text{Debt-Equity Ratio} = \frac{\text{Total Debt}}{\text{Total Equity}}.$$

The **debt ratio** is

$$\text{Debt Ratio} = \frac{\text{Total Debt}}{\text{Total Assets}}.$$

The **equity multiplier** is

$$\text{Equity Multiplier} = \frac{\text{Total Assets}}{\text{Total Equity}}.$$

The **times interest earned ratio** is

$$\text{Times Interest Earned} = \frac{EBIT}{\text{Interest Expense}}.$$

The interest coverage ratio is directly related to the firm's ability to make interest payments.

Return measures relate income to the capital invested in the firm.

Definition 5.16 The **return on equity** is

$$ROE = \frac{\text{Net Income}}{\text{Average Stockholders' Equity}}.$$

When the balance sheet denominators are measured consistently using period averages, the **return on assets** can be decomposed as

$$ROA = \underbrace{\frac{NI}{TOR}}_{\text{Profit Margin}} \cdot \underbrace{\frac{TOR}{ATA}}_{\text{Asset Turnover}},$$

so that the **DuPont decomposition** for return on equity is

$$ROE = \frac{NI}{TOR} \cdot \frac{TOR}{ATA} \cdot \frac{ATA}{ASE} = \text{Profit Margin} \cdot \text{Asset Turnover} \cdot \text{Average Equity Multiplier},$$

where ASE is average stockholders' equity. Mixing average assets with an end-of-period equity multiplier would break the algebraic identity.

The DuPont decomposition is useful because it separates operating efficiency, asset use efficiency, and leverage. A firm with a high return on equity may be earning that return through strong margins, fast turnover, heavy leverage, or some combination of the three.

Finally, market value ratios connect accounting information to security prices.

Definition 5.17 The **price-earnings ratio** is

$$P/E = \frac{\text{Market price per share}}{\text{Earnings per share}}.$$

The **dividend yield** is

$$\text{Dividend Yield} = \frac{\text{Dividend per share}}{\text{Price per share}}.$$

The **market-to-book ratio** is

$$M/B = \frac{\text{Market value per share}}{\text{Book value per share}}.$$

These ratios connect accounting data to market prices. In particular, an M/B ratio below 1 suggests that the market does not value the firm's investments above their accounting cost.

Remark 5.18 A ratio being "high" is not automatically good. Interpretation depends on the type of firm and the economic setting.

Remark 5.19 Profit margin and turnover tend to trade off across industries. Very different firms can both be viable even when their ratio profiles look very different. More generally, ratio analysis does not directly incorporate risk or the timing of cash flows, and the ratios are linked to one another rather than being independent signals.

Statement of Cash Flows

The statement of cash flows helps explain the change in accounting cash over the period. It breaks cash movements into three categories:

1. Cash flow from operating activities.
2. Cash flow from investing activities.
3. Cash flow from financing activities.

Example 5.20 Using the same figures, cash flow from operating activities can be reconstructed from net income, non-cash charges, and working capital adjustments:

$$CFO = 86 + 90 + 13 - 24 + 11 + 16 + 18 - 3 - 8 = 199.$$

Cash flow from investing activities is

$$CFI = -198 + 25 = -173,$$

and cash flow from financing activities is

$$CFF = -73 + 86 - 43 - 6 + 43 = 7.$$

Therefore the net change in cash is

$$\Delta\text{Cash} = 199 - 173 + 7 = 33,$$

which matches the increase in cash and equivalents from 107 to 140.

This accounting statement is not identical to the finance notion of cash flow from assets, but the two are closely related. The finance reconstruction isolates operating cash flow, capital spending, and additions to net working capital because those are the quantities most directly used in valuation.

Lecture 8 — Wednesday, January 22, 2014

Financial Planning and Growth

Financial planning is not an end in itself; the model helps us understand future financing needs and the consequences of management decisions.

A financial plan describes how the firm intends to achieve its financial goals. Two dimensions matter: the **time frame**—usually less than a year for short-run planning and roughly two to five years for long-run planning—and the **level of aggregation**, since the firm-wide plan combines the plans of its divisions and projects.

Scenario analysis is a standard part of this process. A division may be asked to prepare a worst-case plan, a normal-case plan, and a best-case plan. The purpose is not to predict the future with certainty, but to understand how financing needs and operating outcomes change when the underlying assumptions change.

Because investment and financing decisions interact, financial planning is inherently iterative. Firms often plan one year ahead and revisit the model periodically, with the sales forecast driving nearly everything. A standard planning process is:

1. Build a corporate financial model.
2. Describe possible future scenarios.
3. Construct pro forma financial statements.
4. Run the model under alternative scenarios.
5. Examine the strategic implications.

The planning process should make the interaction between investment policy and financing policy explicit, force the firm to confront the available options, check feasibility against the objective of maximizing shareholder wealth, and reduce the risk of unpleasant surprises. In short, the plan should act as a disciplined way of thinking ahead.

Remark 8.1 The model is not the goal; coherent forward-looking decision making is the goal.

A planning model includes:

1. Sales forecast.
2. Pro forma statements.
3. Asset requirements.
4. Financial requirements.
5. A plug variable.
6. Economic assumptions.

Each ingredient serves a specific purpose. The sales forecast anchors the entire model, even though perfect foresight is impossible and the forecast depends on an uncertain macroeconomic and industry environment. Pro forma statements translate those assumptions into forecasted income statements, balance sheets, and statements of sources and uses of cash. Asset requirements identify the projected demands on fixed assets and net working capital, while financial requirements describe how the firm expects to raise funds, including dividend policy, capital structure policy, and any possible security issuance.

The plug variable is what restores compatibility if the planner's assumptions do not fit together automatically. For example, if sales, costs, and assets are all assumed to grow in one way but retained financing grows in another, some third variable must adjust. Economic assumptions complete the model by stating the external environment, including interest rates and related background assumptions.

Remark 8.2 The GIGO principle matters: garbage in, garbage out. A financial plan is only as good as the assumptions used to build it.

Definition 8.3 External financing needed (EFN) is the amount by which projected assets exceed projected financing generated internally under the model assumptions.

EFN is the difference between how much financing the firm will need and how much financing it will generate internally. In spreadsheet terms, it is the "plug" that balances the forecasted statements.

The clean conceptual version is

$$EFN = \text{Projected Assets} - \text{Projected Total Financing}.$$

When the percentage of sales method applies, a standard approximation is

$$EFN = \left(\frac{A^*}{S_0}\right)\Delta S - \left(\frac{L^*}{S_0}\right)\Delta S - mS_1(1-d),$$

where A^* denotes the assets that vary directly with sales, L^* denotes the liabilities that vary directly with sales, m is the net profit margin, d is the dividend payout ratio, S_0 is current sales, and S_1 is projected sales. The first term captures the additional asset investment required for growth, the second term captures spontaneous financing from liabilities that rise with sales, and the last term captures internally generated financing through additions to retained earnings.

The **percentage of sales method** builds a pro forma balance sheet in six steps:

1. Express balance sheet items that vary with sales as percentages of sales.
2. Multiply those percentages by projected sales to obtain forecasted values.

3. For items that do not vary with sales, carry forward the previous balance sheet amount.
4. Compute projected retained earnings from

$$RE_1^{\text{projected}} = RE_0 + NI_1^{\text{projected}} - D_1^{\text{projected}}.$$

5. Add projected assets and projected financing separately, and interpret any difference as the shortfall or surplus.
6. Use the plug to fill EFN.

Example 8.4 Suppose a hypothetical firm expects sales to rise from 20 million to 22 million, a growth rate of 10%. Its profit margin on sales is 10%, and its dividend payout ratio is 50%. Costs therefore rise from 16.970 to 18.67, taxable income rises from 3.03 to 3.33, taxes rise from 1.03 to 1.13, and net income rises from 2.0 to 2.2. Dividends rise from 1.0 to 1.1, so additions to retained earnings also rise from 1.0 to 1.1.

On the balance sheet side, current assets rise from 6 to 6.6, fixed assets rise from 24 to 26.4, short-term debt rises from 10 to 11, and long-term debt rises from 6 to 6.6, all under the percentage of sales assumption. Common stock is held constant at 4, while retained earnings rise from 10 to 11.1. Hence projected assets are 33 and projected total financing is only 32.7, so

$$EFN = 33.0 - 32.7 = 0.3.$$

Therefore the firm needs 0.3 million, that is, 300,000, of external financing.

Example 8.5 A typical spreadsheet workflow is:

1. Increase the sales-driven entries by the planned growth rate, often something simple such as 10%.
2. Construct the pro forma income statement vertically rather than row-by-row across years.
3. Update retained earnings using projected net income less projected dividends.
4. Carry forward non-sales-driven balance sheet items as instructed.
5. Compute
$$EFN = \text{Projected Assets} - \text{Projected Total Financing}.$$
6. Interpret the sign of EFN: positive means external financing is required, while nega-

tive means the firm has excess financing capacity under the model assumptions.

These steps capture the standard mechanics of this type of spreadsheet problem.

Sustainable Growth

The sustainable growth rate is the rate of sales growth that produces $EFN = 0$. If the planned growth rate exceeds this level, the firm must raise additional financing under the model assumptions; if it falls below this level, the firm may have more financing than it needs. This sign change is the key idea in Figure 5.

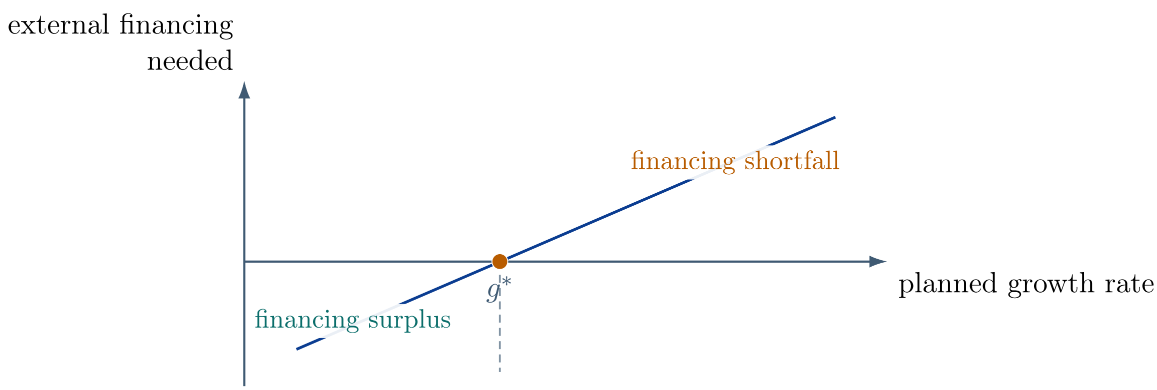

Figure 5

This links growth policy to profitability, payout policy, leverage, and asset intensity. Write $b = 1-d$ for the retention ratio. Under the standard assumptions that the relevant ratios remain constant, the internally financed growth rate is

$$g_{\text{internal}} = \frac{ROA\,b}{1 - ROA\,b},$$

while the sustainable growth rate that preserves the debt–equity ratio without issuing new equity is

$$g_{\text{sustainable}} = \frac{ROE\,b}{1 - ROE\,b}.$$

The sustainable rate is therefore a quantity determined by financial policy, not a purely operational one.

Several actions reduce financing pressure:

1. Issue new shares.
2. Rely more heavily on debt.
3. Reduce the dividend payout ratio.
4. Increase profit margins.
5. Reduce the asset requirement ratio.

Sustainable growth has two practical uses. First, management can compare the actual growth target with the sustainable growth rate to see whether the target is consistent with the firm's current financial policy. Second, lenders can compare a borrower's actual growth rate with its sustainable growth rate. If actual growth is much higher than sustainable growth, the borrower may be "growing broke," meaning that one round of financing is unlikely to be enough.

Sustainable growth can also be negative. If the firm is losing money or paying out more than 100% of earnings as dividends, then the retention ratio becomes negative and sustainable growth can be negative as well. In that case, the interpretation is that sales and assets must shrink rather than expand unless policy changes.

Remark 8.6 Positive planned growth above the sustainable rate means the firm must borrow or issue equity. Negative EFN means the firm has surplus financing capacity rather than a financing shortfall.

Financial planning models do not tell the firm which financial policy is best. They are simplified representations of reality, and reality can change sharply when sales disappoint, financing conditions tighten, or macroeconomic assumptions turn out to be wrong. The planning process is therefore iterative: assumptions are revised, the model is rerun, and the strategic implications are reassessed.

An overly top-down planning culture in which senior management sets a growth target first and then expects the planning staff to force the numbers to fit should be avoided. Such plans can be made to look feasible on paper by building in overly optimistic assumptions, but they collapse when the sales forecast fails to materialize. Financial planning is useful precisely because it exposes these tensions early rather than hiding them.

Lecture 11 — Wednesday, January 29, 2014

3. Time Value and Security Valuation

Financial Markets and Net Present Value

Opportunity cost, arbitrage, intertemporal choice, and the net present value rule connect internal corporate planning to the functioning of financial markets.

Holding cash has an opportunity cost. More generally, if equivalent positions command different prices in two markets, arbitrage becomes possible: buy the cheap side and sell the expensive side.

Individuals and institutions have different income streams and different intertemporal consumption preferences. Because of this, a market for money arises, and the price of money is the interest rate. Someone with excess funds can lend, while someone who wants to spend more today than current income allows can borrow.

A dentist with surplus funds could lend directly to several students who need to borrow for one year. A financial intermediary performs the same function more efficiently by pooling funds and screening borrowers.

Financial intermediation appears in three main forms:

1. **Size intermediation**: One large claim can be transformed into many smaller claims, or vice versa.
2. **Term intermediation**: Short-term funds can be used to finance longer-term claims.
3. **Risk intermediation**: The risk characteristics faced by ultimate lenders and borrowers can be reshaped.

The money market clears when the quantity supplied of loanable funds equals the quantity demanded. The equilibrium price that balances these two sides is the equilibrium interest rate.

An individual can alter consumption across time periods through borrowing and lending. This creates an intertemporal consumption opportunity set that shows the trade-off between consumption today and consumption in the future.

If an individual has endowment (Y_0, Y_1) and can borrow or lend at rate r, then feasible consumption bundles (C_0, C_1) satisfy

$$C_1 = Y_1 + (1+r)(Y_0 - C_0).$$

This line has slope $-(1+r)$. Saving one dollar today increases future consumption by $1+r$, while borrowing one dollar today decreases future consumption by the same amount. Figure 6 shows both movements along the same budget line.

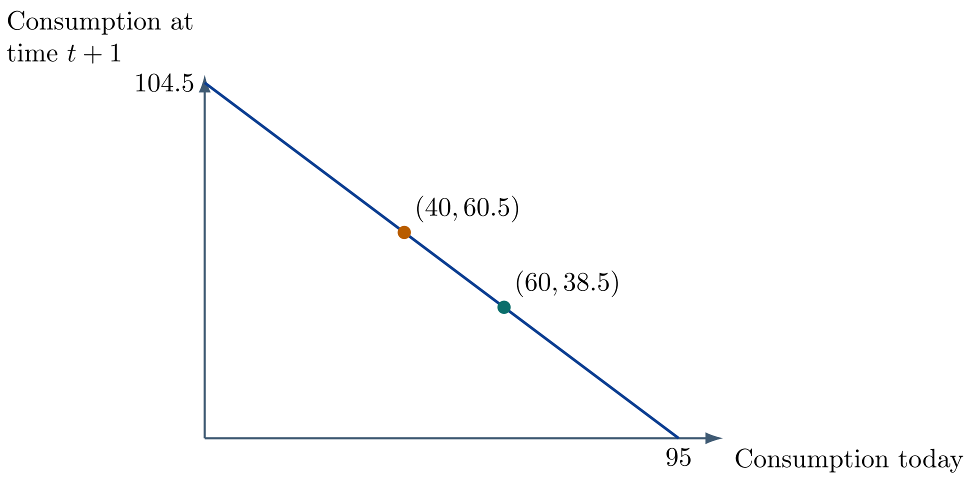

FIGURE 6

Personal preferences determine which point on this line the individual will choose. A relatively patient person will prefer less current consumption and more future consumption, while a relatively impatient person will choose the reverse. A rise in the interest rate rotates the opportunity set, making saving more attractive and borrowing less attractive.

In a competitive market, trading is costless, information about borrowing and lending is available, and there are many traders, so no single participant can move market prices materially. Under these assumptions there can be only one equilibrium interest rate; otherwise arbitrage opportunities would arise.

This single-interest-rate result is essential. If identical borrowing or lending opportunities existed at different prices, an arbitrageur could borrow in the cheap market and lend in the expensive market without bearing genuine economic risk. Competition therefore enforces consistency.

The core investment principle is an opportunity-cost statement:

> An investment must be at least as desirable as the opportunities available in the financial markets.

A project should not be judged in isolation, but relative to what the same money could earn elsewhere at comparable risk.

For example, consider an investment that costs 50,000 today and pays 54,000 next year. Its one-period return is

$$\frac{54{,}000 - 50{,}000}{50{,}000} = 8\%.$$

If the market interest rate is less than 8%, this project dominates the market alternative and should be accepted. If the market rate is above 8%, then the project is inferior to the financial market alternative.

Consider an investor with endowment 40,000 today and 55,000 next year, facing a 10% interest rate. Suppose the investor is offered a project with the cash flow pattern

$$-25{,}000 \text{ at time } 0 \quad \text{and} \quad 30{,}000 \text{ at time } 1.$$

Without the project, the investor's opportunity set is determined entirely by the financial market. With the project, the opportunity set shifts outward if and only if the project has positive net present value, as in Figure 7.

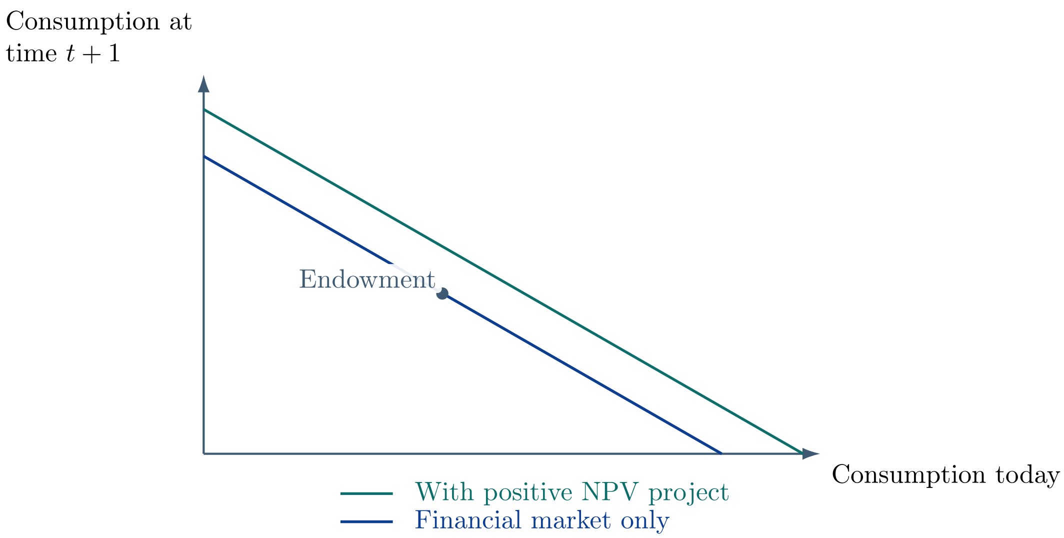

FIGURE 7

The gain can be measured in present value terms. Since the project costs 25,000 today and pays

30,000 next year, its net present value at a market rate of 10% is

$$NPV = -25{,}000 + \frac{30{,}000}{1.10} = 2{,}272.73.$$

This positive net present value is exactly the reason the opportunity set shifts outward. The investor can achieve a higher level of consumption today, next year, or any convex combination in between.

Definition 11.1 The **net present value** of a project is the present value of future cash inflows minus the initial investment.

The decision rule is:

$$\text{NPV} > 0 \implies \text{accept} \quad \text{and} \quad \text{NPV} < 0 \implies \text{reject}.$$

Among competing projects, we prefer the project with the highest net present value, all else being equal.

Positive NPV projects shift the shareholders' opportunity set outward, which is unambiguously beneficial. This is true regardless of whether a particular shareholder is naturally a borrower or a lender, because after the firm has taken the project, each shareholder can still use the financial market to tailor personal consumption over time.

The separation theorem says that all investors will want to accept or reject the same investment projects by using the NPV rule, regardless of their personal preferences. This makes decentralized corporate decision making possible: the firm chooses projects to maximize value, while each shareholder separately borrows or lends to obtain a preferred consumption pattern. Shareholders therefore do not need to vote on every project one by one; managers can use the NPV rule knowing that it aligns with shareholder welfare in competitive capital markets.

Remark 11.2 Drawing timelines is often useful.

Remark 11.3 When decision rules conflict, net present value remains the primary criterion. A project with positive NPV improves the shareholders' feasible consumption set, and that is the foundational reason for the rule.

Lecture 13 — Monday, February 03, 2014

One-Period and Multiperiod Valuation

The standard present value machinery includes accumulation, discounting, annuities, perpetuities, and growing cash flows. The three essential skills are knowing the formulas, converting rates correctly, and laying out the timeline.

The one-period case contains the entire logic of time value of money in its simplest form. If 10,000 is invested for one year at 5%, then the future value is

$$FV = 10{,}000(1.05) = 10{,}500.$$

The additional 500 is interest, while the original 10,000 is repayment of principal.

In general, if C_0 is invested for one period at rate r, then

$$FV = C_0(1 + r).$$

Present value is the reverse calculation. If 10,000 will be received in one year and the market rate is 5%, then the equivalent amount today is

$$PV = \frac{10{,}000}{1.05} = 9{,}523.81.$$

Thus present value is the amount that, if invested today at the market rate, would reproduce the future payment.

Net present value combines cost and benefit. If a claim paying 10,000 in one year can be bought for 9,500 when the market rate is 5%, then

$$NPV = -9{,}500 + \frac{10{,}000}{1.05} = 23.81.$$

Because the net present value is positive, the investment is better than simply placing 9,500 into the market at 5%.

For several periods, the same logic compounds forward or discounts backward repeatedly. The fundamental accumulation formula is

$$FV = C_0(1+r)^T,$$

where C_0 is the cash flow at time 0, r is the effective rate per period, and T is the number of periods. The corresponding present value formula is

$$PV = \frac{C_T}{(1+r)^T}.$$

A dividend-growth example makes the point clearly: if the current dividend is 1.10 and it grows at 40% annually for five years, then the dividend in year five is

$$1.10(1.40)^5 = 5.92.$$

Because compounding is multiplicative, the future value is much larger than it would be if we simply added five copies of the original growth amount.

Sometimes the unknown is the waiting time rather than the value. If 5,000 is invested at 10%, the time required to grow to 10,000 satisfies

$$10{,}000 = 5{,}000(1.10)^T,$$

so

$$T = \frac{\ln(2)}{\ln(1.10)} \approx 7.27 \text{ years}.$$

We can also solve for the required rate. If 5,000 must grow to 50,000 over twelve years, then

$$50{,}000 = 5{,}000(1+r)^{12},$$

which gives

$$r = 10^{1/12} - 1 \approx 0.2115.$$

Thus an annual rate of about 21.15% is required.

Remark 13.1 In practice, there are four standard unknowns in time value of money problems: present value, future value, rate, and time. Once the timeline is clear, every problem is an algebraic rearrangement of the same core identity.

When the cash flow stream is irregular, the cleanest method is to discount each payment separately.

Suppose an investment pays 200 in year 1, 400 in year 2, 600 in year 3, and 800 in year 4, and the discount rate is 12%. Then

$$PV = \frac{200}{1.12} + \frac{400}{1.12^2} + \frac{600}{1.12^3} + \frac{800}{1.12^4} = 1{,}432.93.$$

If the asset is offered for 1,500, it should be rejected because the present value is lower than the cost.

This is a "lumpy" cash flow stream. In such cases, calculator cash flow keys or a spreadsheet are often more convenient than trying to force the stream into a closed form annuity formula.

Compounding and Effective Rates

If compounding occurs m times per year for T years, then

$$FV = C_0 \left(1 + \frac{APR}{m}\right)^{mT}.$$

For example, if 50 is invested for three years at 12% compounded semiannually, then

$$FV = 50 \left(1 + \frac{0.12}{2}\right)^{6} = 70.93.$$

Equivalent nominal rates satisfy

$$\left(1 + \frac{j_m}{m}\right)^{m} = \left(1 + \frac{j_N}{N}\right)^{N}.$$

Continuous compounding appears in the form

$$FV = C_0 e^{rT}.$$

Definition 13.2 The **effective annual rate** (EAR) is the annual rate that gives the same end-of-year wealth as the stated compounding convention.

For an annual percentage rate APR compounded m times per year,

$$EAR = \left(1 + \frac{APR}{m}\right)^{m} - 1.$$

If the stated rate is 12% compounded semiannually, then

$$EAR = \left(1 + \frac{0.12}{2}\right)^2 - 1 = 0.1236.$$

Thus 12% compounded semiannually is equivalent to 12.36% compounded annually.

Example 13.3 An annual percentage rate of 18% compounded monthly has

$$EAR = \left(1 + \frac{0.18}{12}\right)^{12} - 1 \approx 0.1956.$$

Thus the equivalent effective annual rate is 19.56%.

Remark 13.4 The stated annual percentage rate and the effective annual rate are not the same object. The more frequent the compounding, the larger the effective annual rate, holding the stated annual percentage rate fixed.

Annuities, Perpetuities, and Mortgages

Four standard cash flow patterns arise in present value applications:

1. Perpetuity.
2. Growing perpetuity.
3. Annuity.
4. Growing annuity.

For a perpetuity paying C each period at a positive discount rate r,

$$PV = \frac{C}{r}.$$

For a growing perpetuity whose first payment is C and whose growth rate is $g < r$,

$$PV = \frac{C}{r - g}.$$

For an annuity-immediate with payment R and $i \neq 0$,

$$PV = Ra_{\overline{n}|i} = R\left(\frac{1-(1+i)^{-n}}{i}\right),$$

and

$$FV = Rs_{\overline{n}|i} = R\left(\frac{(1+i)^{n}-1}{i}\right).$$

At $i = 0$, both the present and accumulated values are simply nR, which is also the continuous limit of these formulas. For a growing annuity with first payment C, growth rate g, term n, and discount rate $r \neq g$,

$$PV = \frac{C}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^{n}\right].$$

When $r = g$, every discounted payment equals $C/(1+r)$, so the limiting value is $nC/(1+r)$.

The annuity formula can also be understood as the difference between two perpetuities: one beginning at time 1 and one beginning at time $n+1$. This viewpoint is often helpful when deciding where a formula comes from and how to shift it in time.

Example 13.5 If the expected dividend next year is 1.30, dividends grow at 5% forever, and the discount rate is 10%, then

$$P = \frac{1.30}{0.10-0.05} = 26.$$

Example 13.6 If we can afford a monthly car payment of 400 for 36 months and the loan rate is 7% with monthly payments, then the loan amount is the present value of an annuity. Using the monthly rate 0.07/12,

$$PV = 400\left(\frac{1-(1+0.07/12)^{-36}}{0.07/12}\right) \approx 12{,}954.59.$$

Therefore, the most expensive car that can be financed under those assumptions costs about 12,954.59.

Example 13.7 Consider a four-year annuity of 100 per year that makes its first payment two years from today, with discount rate 9%. First value the annuity one period before the first payment:

$$PV_1 = 100\left(\frac{1-1.09^{-4}}{0.09}\right) = 323.97.$$

Then discount that value back one further year:

$$PV_0 = \frac{323.97}{1.09} = 297.22.$$

Therefore, the present value is 297.22.

Example 13.8 If a property pays net rent of 8,500 at the end of the first year, rents grow at 7% annually for 5 years, and the discount rate is 12%, then

$$PV = \frac{8{,}500}{0.12-0.07}\left[1-\left(\frac{1.07}{1.12}\right)^5\right] = 34{,}706.26.$$

A delayed growing annuity stock pricing problem is also common. Suppose dividends are forecast to be 1.50 in year 1, 1.65 in year 2, and 1.82 in year 3, after which dividends grow at 5% forever. If investors require a 10% return, then we first value the stock at time 3:

$$P_3 = \frac{1.82(1.05)}{0.10-0.05} = 38.22.$$

Then the current price is

$$P_0 = \frac{1.50}{1.10} + \frac{1.65}{1.10^2} + \frac{1.82+38.22}{1.10^3} = 32.81.$$

This kind of problem is best handled by drawing the timeline first, locating the first point at which a standard perpetuity or growing-perpetuity formula applies, and only then discounting back to time zero.

Canadian mortgage rates are conventionally quoted as nominal rates compounded semiannually, even when payments are monthly, so we must convert the rate before applying the annuity formulas.

Remark 13.9 Two specifically Canadian features matter. Mortgage rates use semiannual compounding in their quotation convention even though payments are often monthly, and mortgage terms are commonly shorter than the full amortization period.

Example 13.10 Consider a 25-year mortgage of 100,000 at 7.4% nominal compounded semiannually. The effective annual rate is

$$(1 + 0.074/2)^2 - 1 = 0.075369,$$

so the effective monthly rate is

$$i_m = (1.075369)^{1/12} - 1 \approx 0.00607369.$$

With 300 monthly payments, the payment is

$$PMT = 100{,}000 \left(\frac{i_m}{1 - (1 + i_m)^{-300}} \right) \approx 725.28.$$

Therefore, the monthly mortgage payment is approximately 725.28.

Conceptually, a firm is worth the present value of its future cash flows. The hard part is not the time value of money arithmetic itself; it is determining the size, timing, and risk of those cash flows. This is why valuation depends on the same present value logic used throughout the surrounding analysis.

Financial calculators and spreadsheets are useful, but the underlying timeline should always be clear before we press any keys. Level cash flow streams are handled naturally with the time value of money keys, while irregular streams are better handled with a cash flow menu or direct discounting.

Remark 13.11 Do not force closed-form annuity formulas onto a small irregular payment stream. If there are only a few non-level cash flows, discount them one by one and keep the timeline explicit.

Lecture 15 — Friday, February 07, 2014

Valuing Bonds and Stocks

Present-value methods apply directly to the valuation of bonds and common stock.

Definition 15.1 A **bond** is a legally binding agreement between a borrower and a lender specifying the principal amount of the loan together with the size and timing of the promised cash flows. These cash flows may be written directly in dollar terms, as in a fixed-rate bond, or by formula, as in an adjustable-rate instrument.

From first principles, a bond's value is simply the present value of its expected future cash flows. Figure 8 makes the coupon stream and final redemption payment explicit.

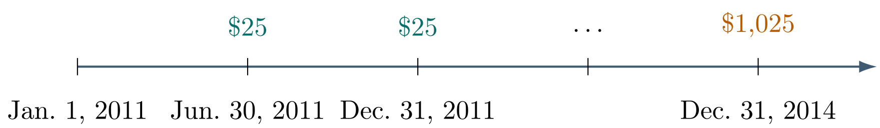

FIGURE 8

This timeline depicts a Government of Canada bond listed as 5.000 December 2014: par value 1,000, semiannual coupons, and coupon payments of 25.

For a pure discount bond with face value F, maturity n, and yield i,

$$PV = \frac{F}{(1+i)^n}.$$

Such bonds make no periodic interest payments, so the entire yield comes from the difference between purchase price and maturity value. Treasury bills and principal-only strips are standard examples.

Example 15.2 A 30-year zero-coupon bond with par value 1,000 and yield to maturity 6% has value

$$PV = \frac{1000}{1.06^{30}} \approx 174.11.$$

Therefore, a long zero-coupon bond can trade at a substantial discount because every dollar of value is received only at maturity.

More generally, a coupon bond is valued by discounting the coupon stream together with the redemption amount. If the coupon payment each period is C, the maturity value is F, there are T periods remaining, and the yield per period is r, then

$$PV = C\left[\frac{1-(1+r)^{-T}}{r}\right] + \frac{F}{(1+r)^T}.$$

Thus a level-coupon bond is just an annuity plus a terminal payment.

Example 15.3 Consider the Government of Canada $6\frac{3}{8}\%$ December 2016 bond on January 1, 2011. The bond has par value 1,000, semiannual coupons of 31.875, and 12 half-year periods remaining. If the required annual yield to maturity is 5%, then the per-period discount rate is 0.025, so

$$PV = 31.875\left[\frac{1-(1.025)^{-12}}{0.025}\right] + \frac{1000}{1.025^{12}} = 1{,}070.52.$$

Therefore, the bond trades at a premium because the coupon rate exceeds the market yield.

A bond sells at a discount when the market yield exceeds the coupon rate, and it sells at a premium when the market yield is below the coupon rate.

If the same $6\frac{3}{8}\%$ bond is instead valued using an annual yield of 11%, then the per-period yield is 5.5% and

$$PV = 31.875\left[\frac{1-(1.055)^{-12}}{0.055}\right] + \frac{1000}{1.055^{12}} = 800.70.$$

Now the bond trades at a discount because the market demands a higher return than the coupon rate supplies.

Remark 15.4 Bond prices and discount rates move in opposite directions. That is not a special theorem about bonds; it is an immediate consequence of present value arithmetic.

Definition 15.5 The **holding period return** is

$$r = \frac{\text{Coupons} + \text{Sell Price} - \text{Initial Purchase Price}}{\text{Initial Purchase Price}}.$$

Example 15.6 Suppose a 6.375% Government of Canada bond maturing December 31, 2018 is purchased on January 1, 2011 for 1,089.75 when the yield is 5%. Six months later, after receiving a coupon of 31.875, the bond is sold when the yield has fallen to 4%. The sale price is

$$31.875\left[\frac{1-(1.02)^{-15}}{0.02}\right] + \frac{1000}{1.02^{15}} = 1{,}152.59.$$

The six-month holding period return is therefore

$$\frac{31.875 + 1{,}152.59 - 1{,}089.75}{1{,}089.75} = 0.0869.$$

This annualizes to an effective annual return of

$$(1.0869)^2 - 1 = 0.1814.$$

Therefore, the effective annual holding period return is 18.14%.

Yield to maturity can also be read in reverse: it is the single discount rate that equates the current bond price to the present value of all remaining promised payments; algebraically, it is the internal rate of return on the bond's cash flow stream.

Bond amortization schedules require attention, especially the sign conventions. The first and last few rows of the schedule deserve particular care.

Year	Face Value	Coupon	Interest	Book Value
0				B_0
1	F	Fr	$B_0 i$	B_1
2	F	Fr	$B_1 i$	B_2

A useful numerical observation: the ratio between consecutive write-ups in a discount-bond amortization schedule is $1+i$, where i is the discount rate.

Remark 15.7 Two habits are worth cultivating: understand the sign convention in the amortization table, and be prepared to rank bonds by riskiness or price sensitivity rather than merely compute a single price.

The key bond pricing relationships now fit together. A bond trades at par when its coupon rate equals its yield to maturity, at a premium when the coupon rate exceeds the yield, and at a discount when the coupon rate falls below the yield. Two comparative statics facts complete the picture: bonds with longer maturities are more sensitive to a change in yield, and bonds with lower coupons are more sensitive as well, all else equal.

A long-maturity bond places more of its value in distant cash flows, so a change in discount rate has more time to operate. Similarly, a low-coupon bond behaves more like a zero-coupon bond: more of its value is concentrated in the final payment, making it more yield-sensitive. Figure 9 also shows the convexity of the price–yield relation and the greater steepness of the longer bond around par.

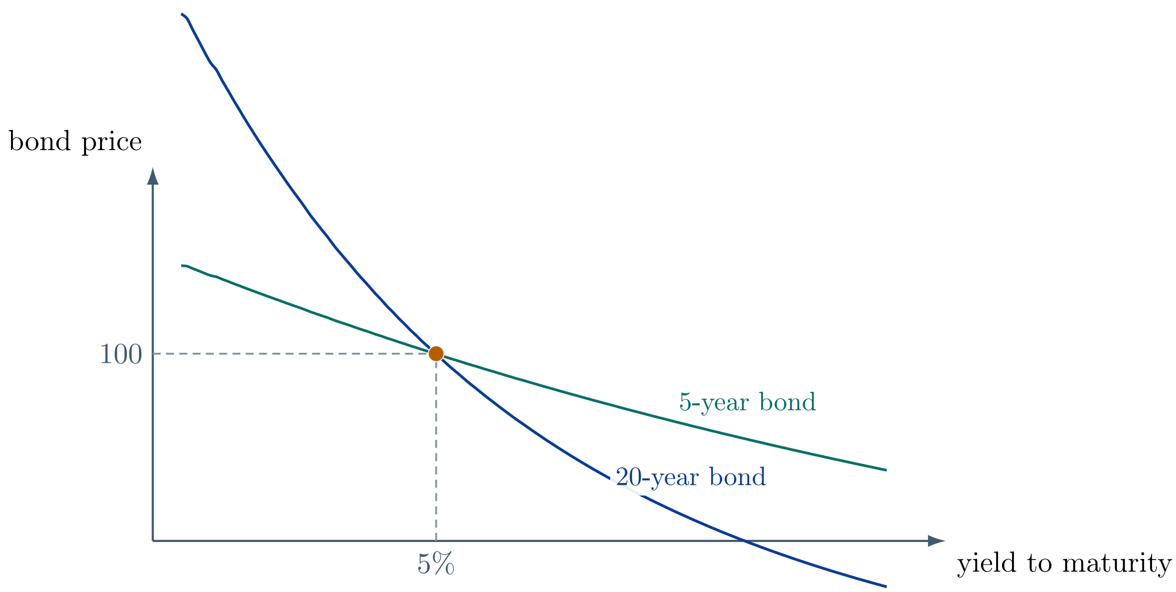

FIGURE 9

Stocks and the Dividend Discount Model

The value of any common stock is the present value of its expected future cash flows to shareholders. Those cash flows come partly as dividends and partly as capital gains. The dividend discount model emphasizes that capital gains are not an independent source of value; they arise because future buyers are themselves willing to pay for later dividends.

Three cases arise in stock valuation: no growth, constant growth, and differential growth.

With no growth, the stock behaves like a perpetuity:

$$P = \frac{Div}{r}.$$

Example 15.8 ABC Corp. is expected to pay a constant dividend of 0.75 per year starting one year from now. If the required return is 12%, then

$$P_0 = \frac{0.75}{0.12} = 6.25.$$

Therefore, a zero-growth stock is valued exactly like a perpetuity.

Under constant growth with $g < r$,

$$P = \frac{Div_1}{r - g},$$

where g is the sustainable growth rate, with

$$g = \text{Retention Ratio} \cdot ROE.$$

Example 15.9 XYZ Corp. is expected to pay 3.60 next year, and dividends are expected to grow at 4% forever. If the required return is 16%, then

$$P_0 = \frac{3.60}{0.16 - 0.04} = 30.00.$$

Therefore, the Gordon growth formula values the stock at 30.

Differential growth applies when a firm grows unusually quickly for a finite period before settling into a stable growth rate. The correct method is to value the high-growth period explicitly and then attach a terminal value at the transition date, following the decomposition in Figure 10.

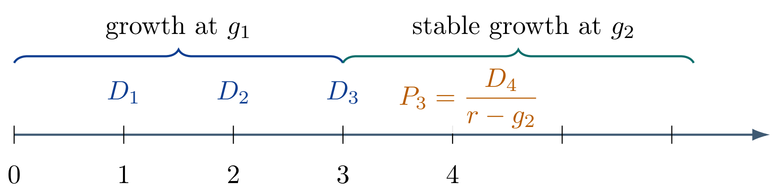

FIGURE 10

Example 15.10 Suppose a common stock has just paid a dividend of 2. Dividends grow at 8% for three years and then at 4% forever, with discount rate 12%. Then

$$D_1 = 2(1.08) = 2.16, \quad D_2 = 2(1.08)^2 = 2.33, \quad \text{and} \quad D_3 = 2(1.08)^3 = 2.52.$$

The first dividend in the stable growth phase is

$$D_4 = 2(1.08)^3(1.04) = 2.62.$$

Hence the terminal value at time 3 is

$$P_3 = \frac{2.62}{0.12 - 0.04} = 32.75.$$

Discounting everything to time 0 gives

$$P_0 = \frac{2.16}{1.12} + \frac{2.33}{1.12^2} + \frac{2.52 + 32.75}{1.12^3} = 28.89.$$

Therefore, the stock is worth 28.89.

A closed-form decomposition for differential growth is also available: value the first N years as a growing annuity at rate g_1, then add the discounted value of a growing perpetuity beginning in year $N+1$ with growth rate g_2. In practice, discounting each cash flow individually is often the safer approach.

Growth and Discount Rates

The formulas above require estimates of both the growth rate g and the discount rate r. A common approximation is

$$g = \text{Retention Ratio} \times \text{Return on retained earnings},$$

with return on retained earnings often proxied by historical return on equity.

Example 15.11 Ontario Book Publishers reports earnings of 1.6 million and plans to retain 28% of earnings. If its historical ROE is 12%, then the expected growth rate is

$$g = 0.28 \times 0.12 = 0.0336.$$

Therefore, expected growth is 3.36%.

For the discount rate, the constant growth formula can be rearranged to obtain

$$r = \frac{Div_1}{P_0} + g.$$

Thus required return is decomposed into dividend yield plus growth rate.

Example 15.12 Manitoba Shipping Co. is expected to pay a dividend next year of 8.06, dividends are expected to grow at 2% forever, and the stock currently sells for 62. Then

$$r = \frac{8.06}{62} + 0.02 = 0.15.$$

Therefore, the market-required return is 15%.

NPVGO and Market Multiples

We can value growth opportunities in two equivalent ways. The first is to use the constant growth dividend discount model directly; the second is to decompose the stock price as

$$P = \frac{EPS}{r} + NPVGO,$$

where $NPVGO$ is the net present value of growth opportunities.

This decomposition is useful because it separates the value of the firm as a pure cash cow from the additional value created by retaining earnings and investing them in positive-NPV projects. Growth by itself is not necessarily valuable; only profitable growth is.

Example 15.13 Consider a firm with $EPS = 5$ at the end of year one, dividend payout ratio 30%, discount rate 16%, and return on retained earnings 20%. The dividend is

$$Div_1 = 5(0.30) = 1.50,$$

the retention ratio is 0.70, and the growth rate is

$$g = 0.70 \times 0.20 = 0.14.$$

The dividend discount model gives

$$P_0 = \frac{1.50}{0.16 - 0.14} = 75.$$

The cash cow value is

$$\frac{EPS}{r} = \frac{5}{0.16} = 31.25,$$

so the implied value of growth opportunities is

$$NPVGO = 75 - 31.25 = 43.75.$$

Therefore, most of the firm's value comes from profitable growth opportunities rather than from simply paying out current earnings.

Remark 15.14 $NPVGO$ can be negative. Growth is only valuable if the reinvested earnings earn a sufficiently high return.

Other useful ratios include the dividend yield, the market-to-book ratio, and the price-earnings ratio. The price-earnings ratio, or multiple, is

$$P/E = \frac{\text{Price per share}}{EPS}.$$

Growth stocks often sell at high multiples, while firms that are out of favour tend to sell at lower multiples. Comparable-company valuation proceeds by averaging observed P/E ratios for similar firms and applying the resulting multiple to the target firm's earnings.

Stock tables then bundle several ideas together: dividend yield, trading volume, 52-week high and low, current close, and daily price change. Reading such a table correctly is part of practical valuation literacy, because it translates valuation formulas into the language used in financial reporting.

Lecture 17 — Wednesday, February 12, 2014

4. Investment Rules and Capital Budgeting

Alternative Investment Rules

Several decision rules are available, but net present value remains the ultimate benchmark whenever they conflict.

For a project with initial investment CF_0 and future cash flows $CF_1, CF_2, \ldots$,

$$NPV = -CF_0 + \frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \cdots .$$

The acceptance rule is simple: accept if $NPV > 0$. The ranking rule is also simple: among feasible alternatives, prefer the project with the highest net present value. This rule matters because accepting a positive-NPV project increases shareholder wealth.

The net present value rule is the benchmark for three reasons. It uses cash flows rather than accounting earnings, includes the entire cash flow stream, and discounts every payment properly. The NPV calculation itself does not require interim cash flows to be reinvested. If we choose to express the project through a terminal-value calculation, compounding interim cash flows at the opportunity cost of capital reproduces the same ranking. The nonconventional stream in Figure 11 is a useful test case because it contains a negative cash flow after the initial outlay.

Tiny positive or negative values should not be trusted blindly; results are sensitive to the underlying assumptions. Still, when the estimates are credible, NPV is the correct rule.

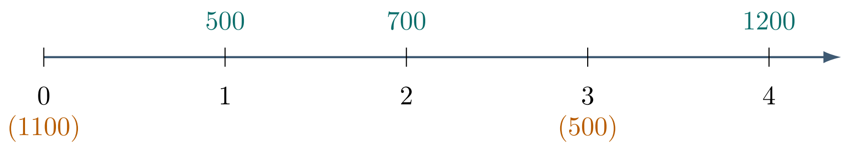

FIGURE 11

Example 17.1 For a project with initial outlay 1,100, discount rate 10%, and cash flows $500, 700, -500, 1200$,

$$NPV = -1100 + \frac{500}{1.1} + \frac{700}{1.1^2} - \frac{500}{1.1^3} + \frac{1200}{1.1^4} = 377.02.$$

Since this is positive, the project is accepted.

Definition 17.2 The **payback period** is the time required to recover the initial investment from undiscounted cash flows.

Management sets the cutoff. The rule is easy to understand and tends to favour liquidity, but it ignores the time value of money, ignores cash flows after the cutoff date, is biased against long-term projects, and may accept projects with negative net present value.

Definition 17.3 The **discounted payback period** is the time required to recover the initial investment after discounting the cash flows first.

It therefore addresses the time value of money objection, but it still discards cash flows occurring after the discounted payback date.

Example 17.4 Suppose a project has initial outlay 1,000, required return 10%, and yearly cash flows $200, 400, 700, 300$. The discounted cash flows are $182, 331, 526, 205$, giving accumulated discounted cash flow totals of

$$182, \quad 513, \quad 1{,}039, \quad \text{and} \quad 1{,}244.$$

Thus the project pays back on a discounted basis just before the end of year three. More precisely,

$$\text{discounted payback} = 2 + \frac{1{,}000 - 513}{526} \approx 2.93 \text{ years.}$$

Definition 17.5 The **average accounting return** is

$$AAR = \frac{\text{Average Net Income}}{\text{Average Book Value of Investment}}.$$

This rule uses accounting quantities rather than cash flows measured at market value. It is easy to compute when accounting statements are readily available, but it ignores the time value of money, uses arbitrary cutoff rates, and relies on book values rather than cash flows.

Example 17.6 A machine costs 90,000, lasts three years, and is depreciated straight-line to zero, so annual depreciation is 30,000. Suppose projected net incomes are -6, 18, and 48 thousand dollars. Then average net income is

$$\frac{-6 + 18 + 48}{3} = 20.$$

The book values over time are $90, 60, 30, 0$, so average book value is

$$\frac{90 + 60 + 30 + 0}{4} = 45.$$

Therefore,

$$AAR = \frac{20}{45} = 44.44\%.$$

The project is accepted if management's target AAR is below 44.44% and rejected otherwise.

Definition 17.7 The **internal rate of return** is the discount rate that makes net present value zero.

The rule says to accept a conventional investment project if IRR exceeds the required return. It is popular because it is easy to communicate as a percentage return. A terminal-value interpretation that compounds interim cash flows at the IRR does impose reinvestment at that rate; the root equation defining IRR itself, however, is simply an equation of value and does not require an actual reinvestment strategy.

Example 17.8 Consider the project with cash flows −200, 50, 100, and 150. The internal rate of return solves

$$0 = -200 + \frac{50}{1+IRR} + \frac{100}{(1+IRR)^2} + \frac{150}{(1+IRR)^3}.$$

The solution is $IRR = 19.44\%$. Thus the project is acceptable whenever the required return is below 19.44%.

Modified internal rate of return uses separate financing and reinvestment rates. It is constructed by discounting outflows to the present at the borrowing rate, compounding inflows to the future at the investment rate, and solving for the single rate that links those two values.

Definition 17.9 For a general cash flow stream, the **profitability index** is

$$PI = \frac{\text{PV of Cash Inflows}}{\text{PV of Cash Outflows}}.$$

For a conventional project with one initial outlay and only later inflows, this reduces to the familiar ratio of the present value of future cash flows to the initial investment.

Accept if $PI > 1$. The rule is easy to understand and can be useful when available investment funds are limited, but it can mis-rank mutually exclusive projects because it scales value by investment size rather than focusing on absolute dollar wealth creation.

Three standard problems arise with internal rate of return:

1. Multiple internal rates of return when cash flow signs change more than once.
2. The scale problem.
3. The timing problem.

The **crossover rate** is the internal rate of return of the difference project $A - B$. It is useful when comparing mutually exclusive projects.

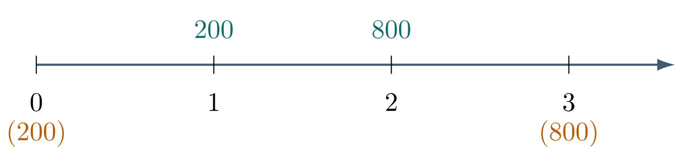

Figure 12

This pattern with multiple sign changes produces two internal rates of return. Once cash flow signs alternate more than once, the internal rate of return rule becomes ambiguous.

The standard example has cash flows $-200, 200, 800, -800$, shown in Figure 12. Its NPV profile in Figure 13 crosses the horizontal axis twice, once at 0% and once at 100%. A project cannot have two economically meaningful internal rates under a single decision rule, so this case exposes a genuine defect of the IRR criterion rather than a mere computational inconvenience.

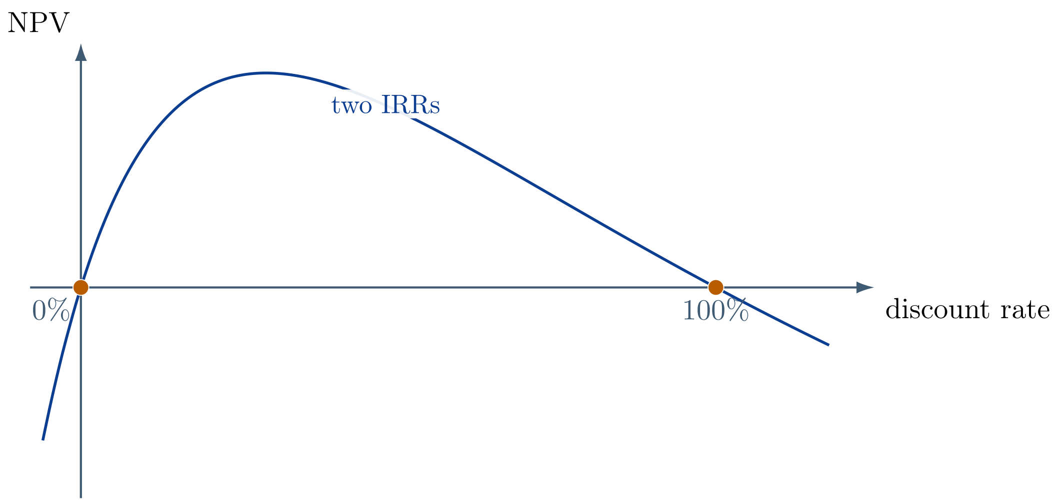

FIGURE 13

The standard criticisms of internal rate of return can be organized around four issues:

1. The possibility of multiple internal rates of return.
2. The distinction between borrowing and lending.
3. The scale of the project.
4. The timing of the cash flows.

The scale problem arises because a project with a very high percentage return on a tiny investment may create much less wealth than a larger project with a lower percentage return. The timing problem arises because mutually exclusive projects may deliver cash flows at different times, causing their NPV profiles to cross.

> **Remark 17.10** Modified internal rate of return addresses two of these concerns directly: it yields a single answer and uses explicit borrowing and reinvestment rates.

Remark 17.11 Independent projects and mutually exclusive projects should be treated differently. For independent projects, each project must exceed the minimum acceptance criterion. For mutually exclusive projects, we rank the alternatives and choose the best feasible project.

The crossover rate is the internal rate of return of the difference project $A - B$ or $B - A$. It identifies the discount rate at which two mutually exclusive projects have the same net present value. Below that rate one project is preferred; above it the other is preferred. This is why NPV profiles are more informative than IRR numbers alone when project timing differs substantially.

Practice varies by industry and by firm size. Some firms still rely heavily on payback or accounting rate of return rules, especially smaller firms or managers who emphasize simplicity and liquidity. Large Canadian industrial corporations use discounted cash flow methods such as NPV and IRR much more frequently.

Example 17.12 Consider two projects with required return 10%:

$$A = (-200, 200, 800, -800) \quad \text{and} \quad B = (-150, 50, 100, 150).$$

A summary of the results gives

$$NPV_A = 41.92, \quad PI_A = 1.0523, \quad \text{and} \quad IRR_A = 0\% \text{ and } 100\%,$$

and

$$NPV_B = 90.80, \quad PI_B = 1.6053, \quad \text{and} \quad IRR_B = 36.19\%.$$

For project A, the profitability index treats the time-3 outflow as an outflow rather than netting it against the earlier inflows. Project B has payback period 2 years, while the ordinary payback rule reports only 1 year for project A: cumulative cash flow first reaches zero at year 1, and the rule then ignores the later 800 outflow. This compact example shows why IRR, profitability index, and payback require special care once cash flow signs are nonconventional.

Ultimately, the alternative rules are useful mainly as supplements. They may help organize thinking about liquidity, accounting performance, or capital rationing, but when they disagree with NPV, finance theory gives priority to net present value.

Lecture 20 — Wednesday, February 26, 2014

NPV and Capital Budgeting

Net present value applies directly to realistic project analysis. The key ingredients are incremental after-tax cash flows, working capital, capital spending, tax shields, and replacement logic.

Incremental After-Tax Cash Flows

Definition 20.1 Incremental after-tax cash flow is the change in the firm's after-tax cash flow caused by undertaking the project.

A small set of principles governs essentially every capital budgeting problem. Cash flows, rather than accounting earnings, are the relevant quantities. Sunk costs have already been incurred and therefore do not matter, whereas opportunity costs do matter because undertaking one project may require giving up another valuable use of the same resources. Side effects also enter the calculation: cannibalism and erosion reduce incremental cash flows, while synergies increase them. Finally, valuation uses after-tax cash flows, and inflation must be treated consistently in both the cash flows and the discount rate.

The main project value decomposition is

$$\begin{aligned} NPV(\text{Project}) = {} & PV(\text{after-tax operating cash flows}) + PV(\text{CCA tax shields}) \\ & + PV(\text{after-tax salvage}) + PV(\text{NWC recoveries}) \\ & - PV(\text{capital expenditures}) - PV(\text{NWC investments}). \end{aligned}$$

The timeline in Figure 14 keeps these components in their proper periods.

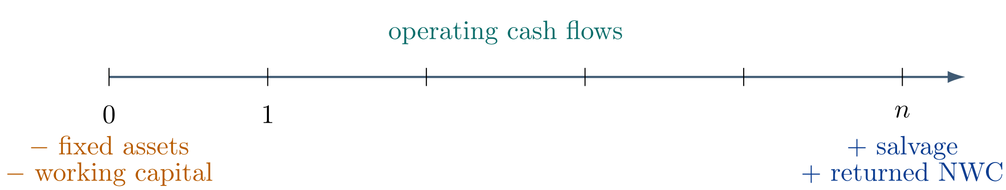

FIGURE 14

This timeline is the basic template behind most project problems. At time zero, the firm typically pays for fixed assets and invests in working capital; during the middle years, it receives operating cash flows; and in the final year, it often receives both salvage proceeds and a recovery of working capital.

> **Remark 20.2** Depreciation or CCA is not itself a cash flow. It matters only because it reduces taxable income and therefore creates a tax shield.

Consider a hypothetical compost producer evaluating a new product line. The firm has already spent 250,000 on test marketing, so that amount is sunk. The proposed site has no resale value, making its opportunity cost zero. The machine costs 800,000 and belongs to Class 8, with a 20% CCA rate; projected production over its eight-year life is

$$6{,}000, 9{,}000, 12{,}000, 13{,}000, 12{,}000, 10{,}000, 8{,}000, 6{,}000.$$

The price in year one was 100 and grew at 2% per year, while variable production cost began at 64 per unit and grew at the inflation rate of 5%. Fixed production cost was 50,000 per year. Working capital was 40,000 initially and then 15% of sales at each year-end, before falling back to zero at the end of the project.

The example brings the full capital budgeting workflow together: we forecast revenues and operating costs, compute CCA and taxes, track working capital changes, and then discount the resulting incremental after-tax cash flows.

> **Example 20.3** In year five, sales revenue is obtained from units times price:
>
> $$12{,}000\left[100(1.02)^4\right] = 1{,}298{,}919.$$
>
> Year-five production cost is variable cost plus fixed cost:
>
> $$12{,}000\left[64(1.05)^4\right] + 50{,}000 = 983{,}509.$$
>
> If the opening UCC in year five is 368,640, then the year-five CCA charge is
>
> $$0.20(368{,}640) = 73{,}728.$$
>
> This is exactly the kind of table-driven workup needed in practice.

Once all pieces are assembled, the project's incremental after-tax cash flow stream is

$$-840{,}000, 81{,}600, 167{,}820, 207{,}564, 237{,}056, 230{,}835, 195{,}175, 148{,}228, \text{ and } 359{,}570.$$

At discount rates of 4%, 10%, 15%, and 20%, the corresponding NPVs are approximately

$$500{,}135, \quad 188{,}042, \quad 2{,}281, \quad \text{and} \quad -137{,}896,$$

with internal rate of return approximately 15.07%. The message is that project attractiveness can change materially with the discount rate even when the operating forecasts are fixed.

Assuming that project net income contains no separately allocated interest expense, three equivalent formulas for operating cash flow are

$$OCF = NI + \text{Depreciation},$$
$$OCF = \text{Sales} - \text{Cash costs} - \text{Taxes},$$
$$\text{and} \qquad OCF = (\text{Sales} - \text{Cash costs})(1 - T_c) + \text{Depreciation}\, T_c.$$

These are the bottom-up, top-down, and tax shield approaches, respectively. The bottom-up route begins with net income, the top-down route stays entirely in cash flow terms, and the tax shield route makes the role of depreciation or CCA especially transparent.

Remark 20.4 Interest expense is excluded from project operating cash flow because financing is handled through the discount rate. Mixing interest expense into the project cash flows would double count financing effects.

Inflation is not a side issue. It changes both the cash flows and the discount rate, so nominal and real quantities must not be mixed carelessly. The relation between real and nominal rates can be summarized using the Fisher relationship:

$$1 + i_{\text{nominal}} = (1 + i_{\text{real}})(1 + \pi).$$

For low inflation,

$$i_{\text{real}} \approx i_{\text{nominal}} - \pi.$$

The practical rule is simple: discount nominal cash flows at nominal rates, or discount real cash flows at real rates. Either approach is acceptable if it is applied consistently.

CCA and Tax Shields

Canadian capital cost allowance is used rather than straight-line depreciation. The standard worksheet tracks beginning undepreciated capital cost, CCA, ending undepreciated capital cost, tax shields, and the present values of those tax shields.

Year	Beginning UCC	CCA	Ending UCC	Tax Shield
1	B_1	$C_1 = dB_1/2$	$E_1 = B_1 - C_1$	T_cC_1
2	$B_2 = E_1$	$C_2 = dB_2$	$E_2 = B_2 - C_2$	T_cC_2

This table shows the simple case in which B_1 is the cost of one new acquisition, the half-year rule applies, and the class contains no other transactions. The half-year rule reduces the first-year CCA deduction on that acquisition. When its assumptions are satisfied, a closed form tax shield calculation can incorporate both that adjustment and the eventual sale of the asset; this avoids discounting every annual shield separately and permits a different discount rate for the tax shield when appropriate.

Remark 20.5 Non-profit organizations do not pay tax, so there is no tax shield in that setting.

A simple but important separation is worth repeating: depreciation is not itself a cash flow, but it affects cash flow through the tax shield it creates.

Many project problems involve an initial outlay followed by annuity-like after-tax cash flows, together with salvage value and working capital adjustments. Writing down the net present value equation first clarifies the structure before any calculation begins.

Example 20.6 A typical project may have the structure

$$\text{NPV} = \left[(2500(1-0.25) - 1000)\,(1-0.4)\right] a_{\overline{5}|0.1} - 3000 + 1105.92(1.1)^{-5}.$$

The important structure is after-tax periodic cash flow, annuity factor, initial outlay, and discounted salvage value.

Remark 20.7 Write the net present value equation before touching the calculator: this is the easiest way to keep operating cash flow, CCA shields, salvage, and working capital adjustments from being mixed together.

Example 20.8 Suppose the existing business would have produced cash flows $100, 120, 100, 150, 500$, but the new project reduces them by $10, 10, 20, 30, 100$. Those reductions are negative incremental project cash flows; using the new project's revenue in isolation would miss the cannibalization entirely.

When projects have unequal lives, we can either repeat the shorter project or compute an equivalent annual cost.

Equivalent Annual Cost

The unequal-life problem arises when projects provide the same essential service but last for different lengths of time. In that setting, comparing raw NPVs can be misleading because one project may need to be replaced sooner.

Suppose a firm must choose between two air cleaners that provide the same legally required service:

- The Cadillac cleaner, which costs 4,000 today, has annual operating cost 100 and lasts 10 years.
- The cheaper cleaner, which costs 1,000 today, has annual operating cost 500 and lasts 5 years.

At first glance, the cheaper cleaner seems preferable because its standalone NPV is less negative:

$$NPV_{\text{Cadillac}} = -4{,}614.46 \quad \text{and} \quad NPV_{\text{Cheap}} = -2{,}895.39.$$

But this is not a fair comparison because the cheaper cleaner must be replaced sooner.

The equivalent annual cost method converts each project's NPV into the level annual cost that has the same present value. If

$$NPV = EAC \cdot a_{\overline{n}|r},$$

then

$$EAC = \frac{NPV}{a_{\overline{n}|r}}.$$

Example 20.9 At 10%, the Cadillac cleaner has

$$EAC = \frac{-4{,}614.46}{a_{\overline{10}|0.10}} = -750.98.$$

The cheaper cleaner has

$$EAC = \frac{-2{,}895.39}{a_{\overline{5}|0.10}} \approx -763.80.$$

Since the Cadillac cleaner has the smaller annual equivalent cost in absolute terms, it is the better choice despite the larger initial outlay.

Three related approaches are available: the replacement-chain method, the matching-cycle method, and the equivalent annual cost method. The equivalent annual cost approach is generally the cleanest and most robust.

Remark 20.10 Equivalent annual cost is the right tool when comparing projects with unequal lives that provide the same service.

Lecture 23 — Wednesday, March 05, 2014

Project Risk and Real Options

Most project problems above were deterministic. Uncertainty changes the capital budgeting decision through risk.

A fundamental difficulty in capital budgeting is that future outcomes are uncertain and decisions often occur in sequence rather than all at once. Decision trees force us to identify when information arrives, which choices remain available, and how project value changes as uncertainty resolves. We use squares for decision nodes and circles for chance nodes, then work backward from the terminal values: take expectations at chance nodes and choose the better branch at decision nodes.

Consider a hypothetical pharmaceutical firm that can spend 1 billion today on a one-year testing phase. Test success has probability 60%. If tests succeed, the firm can then invest 1.6 billion in full-scale production for four years; if the tests fail, it will not proceed. Figure 15 makes the timing of information and investment explicit.

Example 23.1 If full-scale production follows a successful test, the annual after-tax cash flow is 1,588, so the date-1 value of production is

$$NPV_1^{\text{success}} = -1{,}600 + 1{,}588 a_{\overline{4}|0.10} = 3{,}433.75.$$

If the test is unsuccessful, the analogous date-1 value is

$$NPV_1^{\text{failure}} = -1{,}600 + 475 a_{\overline{4}|0.10} = -94.31.$$

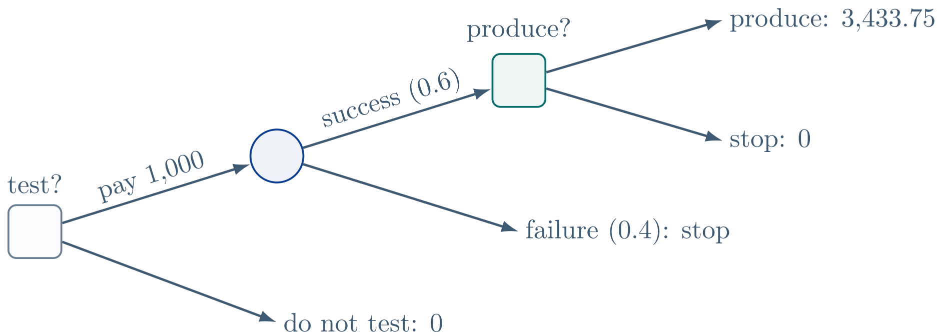

FIGURE 15

Therefore, the firm would invest only after a successful test. The expected continuation value at date 1 is

$$0.6(3{,}433.75) + 0.4(0) = 2{,}060.25.$$

Discounting back to date 0 and subtracting the initial test cost gives

$$NPV_0 = -1{,}000 + \frac{2{,}060.25}{1.10} = 872.95.$$

Therefore, the firm should undertake the test phase.

Sensitivity analysis changes one variable at a time and studies its effect on project value. Scenario analysis changes several variables at once to describe coherent states of the world, such as best-case, normal-case, and worst-case conditions.

For the pharmaceutical project, NPV is highly sensitive to revenue. A substantial reduction in projected revenue produces a much larger proportional fall in NPV, indicating that revenue risk is central to the project's valuation.

Break-even analysis identifies three levels:

1. Accounting break-even.
2. Cash break-even.
3. Financial break-even.

Financial break-even is the sales level at which the project's net present value is zero.

Example 23.2 For the pharmaceutical project, the production stage investment of 1,600 and discount rate 10% imply a break-even level annual incremental after-tax cash flow of

$$IATCF_{BE} = \frac{1{,}600}{a_{\overline{4}|0.10}} = 504.75.$$

Working backward through the income statement gives break-even revenue of approximately 5,358.72. If the original sales plan was 700 million doses, this corresponds to a break-even price of roughly 7.66 per dose.

Scenario analysis is richer because several variables move together. For the pharmaceutical project, we can imagine unusually severe cold seasons with strong sales, normal seasons with base case sales, and mild cold seasons with disappointing demand; each scenario produces its own net present value.

Monte Carlo simulation goes a step further by specifying distributions and interactions among the uncertain variables and then simulating many possible outcomes. It is conceptually useful but not pursued here in a computational way.

The central idea is that a project can have value beyond its static net present value because management has flexibility. The three main real options are to expand, delay, and abandon.

Definition 23.3 The **managerial net present value** of a project is

$$M = NPV + \text{Value of Options}.$$

The **option to expand** has value when success in an initial stage would justify later scaling up the project. Suppose a restaurant chain is considering a test location whose static NPV is

$$NPV = -30{,}000 + 2{,}550a_{\overline{4}|0.10} = -21{,}916.84,$$

so static DCF analysis alone would reject it. But the project may still be worth undertaking because success at the test location creates a valuable option to expand nationally.

The **option to delay** is valuable when project economics are improving over time or when waiting reveals useful information. In a timing example, the present value of benefits at launch remains fixed at 25,000, but launch costs fall over time, so launch-date NPV increases. After discounting those launch-date values back to today, the best launch date is year 2, not immediate launch.

The **option to abandon** matters when downside states allow management to salvage some value rather than simply endure the worst outcome.

The abandonment decision in Figure 16 replaces the zero payoff after failure with the rig's salvage value.

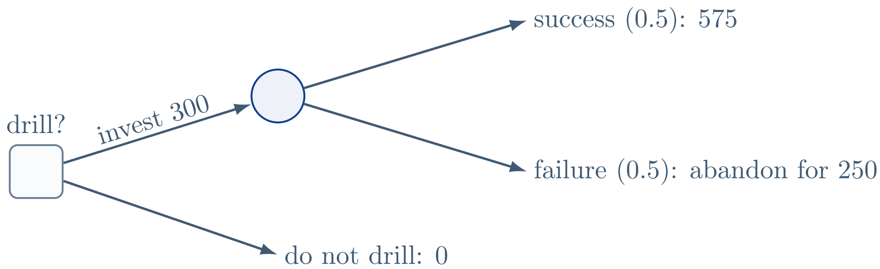

FIGURE 16

Example 23.4 Suppose an oil-drilling rig costs 300 today. In one year the well is either successful or unsuccessful with equal probability. The successful payoff has date-1 value 575, while failure has value 0 if the firm simply stops.

Without abandonment, expected date-1 payoff is

$$0.5(575) + 0.5(0) = 287.50,$$

so static NPV is

$$-300 + \frac{287.50}{1.10} = -38.64.$$

If the firm can sell the rig for 250 after failure, expected date-1 payoff becomes

$$0.5(575) + 0.5(250) = 412.50,$$

and NPV becomes

$$-300 + \frac{412.50}{1.10} = 75.00.$$

Therefore, the option to abandon adds enough value to make drilling worthwhile.

Remark 23.5 The primary task with real options is to recognize when an option exists and to understand that it can alter the capital budgeting decision.

Sensitivity analysis, scenario analysis, break-even analysis, decision trees, and real options all serve the same broader purpose: they help the analyst understand where project value comes from and how management can respond when outcomes differ from the base case forecast.

Lecture 26 — Wednesday, March 12, 2014

5. Options and Corporate Securities

Options and Corporate Finance

Careful distinctions are required between terminology, payoff diagrams, profit diagrams, and valuation methods. Many corporate finance problems can also be reinterpreted as option problems.

Definition 26.1 An **option** gives its holder the right, but not the obligation, to transact in an underlying asset at terms fixed today. A **call option** gives the holder the right to buy the asset at the strike price X, while a **put option** gives the holder the right to sell the asset at X.

Definition 26.2 The buyer of an option takes the **long position** and pays the premium up front. The seller, or writer, takes the **short position** and accepts the obligation that may arise if the buyer chooses to exercise. The **strike price** is the price specified in the contract, and the **expiry date** is the last date on which the contract can remain alive.

Definition 26.3 A **European option** can be exercised only at expiry, whereas an **American option** can be exercised at any time up to expiry.

Unless otherwise stated, European-style notation is used because the valuation logic is easier to present there.

Remark 26.4 Phrases such as "long a call," "short a put," and "exercise the option" should be translated mentally into cash flow rights and obligations before any algebra is attempted.

Definition 26.5 A call option is **in the money** when $S_T > X$, **at the money** when $S_T = X$, and **out of the money** when $S_T < X$. For a put option, the inequalities are reversed.

Definition 26.6 The **intrinsic value** of a call is $\max(S_0 - X, 0)$, and the **intrinsic value** of a put is $\max(X - S_0, 0)$. If the option premium is denoted by V_0, then the **time value** is

$$\text{Time value} = V_0 - \text{Intrinsic value}.$$

This decomposition matters because an in-the-money option can still trade for more than its immediate exercise value when there is time remaining before expiry. For a call, Figure 17 makes the three moneyness regions explicit.

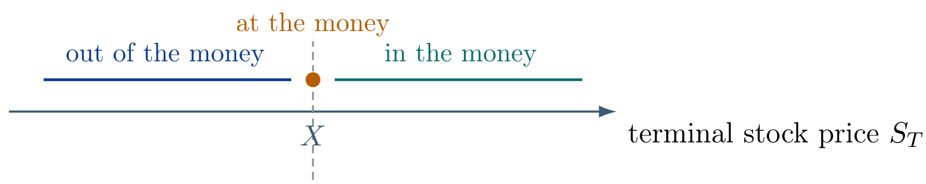

FIGURE 17

Payoff diagrams and profit diagrams are not the same object. The payoff diagram describes what the contract pays at expiry, whereas the profit diagram adjusts the payoff by the premium paid or received initially. Figure 18 places the two basic long-option payoffs side by side.

FIGURE 18

At expiry, the value of a call is

$$C_T = \max(S_T - X, 0),$$

while the value of a put is

$$P_T = \max(X - S_T, 0).$$

For the short positions, the payoffs are simply the negatives of these functions. Under the common diagramming convention that ignores the financing cost of the premium, if the option premium is

c_0 for a call and p_0 for a put, then the profit functions for the long positions are

$$\Pi_{\text{long call}} = \max(S_T - X, 0) - c_0$$

and

$$\Pi_{\text{long put}} = \max(X - S_T, 0) - p_0.$$

The seller's profit is the negative of the buyer's profit. For a fully time-consistent expiry-date profit, the initial premium should instead be accumulated to expiry.

Remark 26.7 The cleanest workflow is to sketch the payoff first and then shift the entire graph vertically by the premium to obtain profit.

The seller of an option receives the premium immediately but accepts a contingent liability. A short call has limited upside, namely the premium received, but potentially very large losses if the stock price rises sharply. A short put also has limited upside, again equal to the premium, but can suffer large losses if the stock price falls substantially.

When the strike price is 50 and the option premium is 10, the long call breaks even at $S_T = 60$, while the short call breaks even at the same point from the opposite side. Similarly, a long put with strike 50 and premium 10 breaks even at $S_T = 40$. These break-even values do not come from a new theory; they come from setting profit equal to zero.

An option quote becomes readable once we identify each field. In the sample quotation, the stock price is 9.35, the contract is a June call with strike 8, the bid price is 1.95, the ask price is 2.10, the daily trading volume is 15 contracts, and the open interest is 660 contracts.

These numbers should be interpreted carefully. Because the stock price 9.35 exceeds the strike price 8, the call is in the money by 1.35. Because listed equity options usually cover 100 shares, buying one contract at the ask costs $2.10 \times 100 = 210$, while selling one contract at the bid yields $1.95 \times 100 = 195$. Volume records how many contracts traded that day, while open interest records how many contracts remain outstanding.

Option Strategies

Puts and calls act as building blocks for more elaborate payoff patterns. Combining them lets us tailor a portfolio's downside protection, upside participation, or volatility exposure; Figure 19 compares three basic shapes on a common scale.

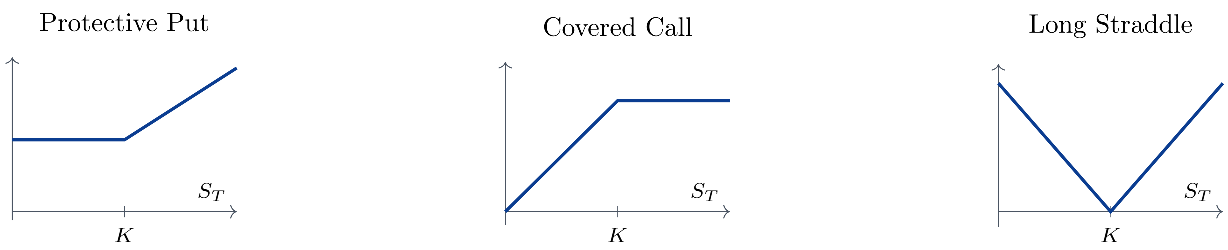

FIGURE 19

Definition 26.8 A **protective put** combines one share of stock with a long put. A **covered call** combines one share of stock with a short call. A **long straddle** combines a long call and a long put with the same strike and expiry. A **bull call spread** combines a purchased call at a lower strike with a written call at a higher strike.

A protective put keeps upside potential while placing a floor under terminal value. A covered call earns premium income but gives up upside beyond the strike price.

A long straddle combines a long call and a long put with the same strike and expiry. If each option costs 10 and the strike is 50, the total premium paid is 20, so the strategy profits only when the stock price finishes below 30 or above 70. The short straddle is the opposite position: it receives 20 initially and loses money only when the stock price moves far away from the strike. This makes the short straddle a bet against volatility.

A bull call spread can be created by buying a call with strike 50 for 10 and selling a call with strike 55 for 7. The net initial cost is therefore 3. At expiry, the payoff is zero if $S_T \leq 50$, rises one-for-one between 50 and 55, and is capped at 5 once $S_T \geq 55$. Ignoring the financing cost of the premiums, the break-even stock price is 53 and the maximum profit is 2. This strategy is appropriate when the investor is moderately bullish but does not want to pay for unlimited upside.

Remark 26.9 A protective put is primarily insurance, whereas a covered call is primarily an income strategy. A long straddle is a volatility trade, while a bull call spread is a limited-upside directional trade.

Put–Call Parity and Synthetic Positions

For European options on a non-dividend-paying stock, with a common strike E, expiry T, and an effective risk-free rate r over the chosen time unit, put–call parity is one of the central arbitrage identities in options:

$$p_0 + S_0 = c_0 + \frac{E}{(1+r)^T}.$$

It says that a protective put has the same terminal payoff as a fiduciary call, meaning a long call plus a risk-free bond that matures to the strike price. The two portfolios trace the same kinked payoff in Figure 20.

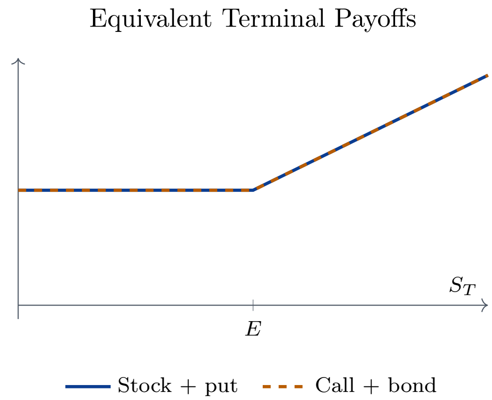

FIGURE 20

Definition 26.10 Under these assumptions, **put–call parity** is the arbitrage identity

$$p_0 + S_0 = c_0 + \frac{E}{(1+r)^T},$$

which says that a protective put has the same terminal payoff as a fiduciary call.

Rearranging gives several synthetic-position formulas. For example,

$$S_0 = c_0 + \frac{E}{(1+r)^T} - p_0.$$

Thus stock can be replicated by a long call, a long risk-free bond, and a short put. Whenever two portfolios have identical terminal payoffs, arbitrage requires them to have identical values today.

The earlier payoff formulas describe values at expiry only. Before expiry, the option has time value, so the current premium depends on several variables. In a no-dividend setup, the main determinants are the current stock price, the strike price, the interest rate, the volatility of the stock price, and the time remaining to expiry. In broader market practice, dividend yield also matters.

The directional effects are intuitive once the rights embedded in the contract are clear. A higher stock price raises call values and lowers put values. A higher strike price lowers call values and raises put values. A higher interest rate tends to raise call values because the cash outlay associated with exercise is postponed, but it tends to lower put values because waiting to sell the asset becomes more costly. Greater volatility increases both call and put values in the standard model because the holder benefits from favorable extreme outcomes while retaining limited downside. Extra time cannot reduce the value of an American option because it enlarges the exercise opportunities. The maturity effect for a European option is not universally positive, however; in particular, a European put can lose value as expiry moves farther away.

For an American call on a non-dividend-paying stock, the basic bounds are

$$\max(S_0 - E, 0) \leq C_0 \leq S_0.$$

The lower bound is the intrinsic value, and the difference between the premium and intrinsic value is the time value.

Binomial Option Pricing

In the one-period binomial model, a stock currently worth 25 either rises by 15% to 28.75 or falls by 15% to 21.25. Consider a one-period call with strike price 25. Its terminal payoffs are 3.75 in the up state and 0 in the down state, as shown alongside the stock tree in Figure 21.

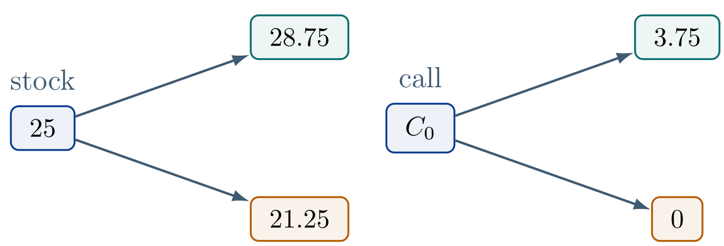

FIGURE 21

Replication comes first. The hedge ratio is

$$\Delta = \frac{3.75 - 0}{28.75 - 21.25} = 0.5.$$

Thus one call can be replicated by holding one-half share of stock and borrowing an amount that will grow to 10.625 next period. At a risk-free rate of 5%, the amount borrowed today is

$$B_0 = \frac{10.625}{1.05} = 10.12.$$

Therefore, the option value is

$$C_0 = 0.5(25) - 10.12 = 2.38.$$

The same answer is obtained by risk-neutral valuation. The risk-neutral probability of an up move is

$$q = \frac{(1+r_f)S_0 - S_d}{S_u - S_d} = \frac{1.05(25) - 21.25}{28.75 - 21.25} = \frac{2}{3}.$$

Hence

$$C_0 = \frac{q(3.75) + (1-q)(0)}{1.05} = \frac{(2/3)(3.75)}{1.05} = 2.38.$$

Remark 26.11 The important lesson is not the arithmetic itself. The important lesson is that a derivative can be valued by finding a portfolio of primitive securities with identical payoffs.

The same example also introduces Δ-hedging. The call has positive delta because its value rises when the stock price rises. A put has negative delta because its value moves in the opposite direction.

Black–Scholes Model

The Black–Scholes model extends the same no-arbitrage logic to a continuous-time setting. For a European call on a non-dividend-paying stock,

$$C_0 = S_0\Phi(d_1) - Ee^{-rT}\Phi(d_2),$$

where

$$d_1 = \frac{\ln(S_0/E) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}.$$

Here r is the continuously compounded risk-free rate, σ is volatility, and $\Phi(x) = \mathbb{P}(Z \le x)$ for $Z \sim \mathcal{N}(0,1)$.

Example 26.12 Consider a six-month call option on a non-dividend-paying stock with current price $S_0 = 160$, strike price $E = 150$, continuously compounded risk-free rate $r = 5\%$, time to expiry $T = 0.5$, and volatility $\sigma = 30\%$. The intrinsic value is already $160 - 150 = 10$, so the option must be worth at least that much.

Compute
$$d_1 = 0.52815 \quad \text{and} \quad d_2 = 0.31602.$$
Hence
$$\Phi(d_1) = 0.7013 \quad \text{and} \quad \Phi(d_2) = 0.62401.$$
Therefore,
$$C_0 = 160(0.7013) - 150e^{-0.05(0.5)}(0.62401) = 20.92.$$
The gap between 20.92 and the intrinsic value 10 is the option's time value.

Corporate securities can often be interpreted as option positions written on firm value. In a levered firm, equity can be viewed as a call option on the value of the firm's assets, with strike price equal to the promised payment to bondholders. If firm assets exceed the debt at maturity, shareholders effectively pay off the debtholders and keep the residual. If firm assets are worth less than the debt, shareholders can walk away, leaving the firm to the debtholders.

The same idea can also be described in put language. If the firm is worth less than the debt at maturity, shareholders effectively put the firm to bondholders by declaring bankruptcy. These two viewpoints are consistent because they are tied together by put–call parity.

Option language clarifies several agency problems of debt. When shareholders hold a call-like claim on firm value, they benefit from higher volatility even if total firm value falls. This is the classic risk-shifting problem.

Example 26.13 Consider a distressed firm whose assets have market value 200 and whose outstanding debt promises 300. If the firm is liquidated immediately, bondholders receive 200 and shareholders receive nothing.

Suppose shareholders can instead invest the firm's 200 cash in a gamble that pays 1000 with probability 0.10 and 0 with probability 0.90. If the required return is 50%, the project's NPV for the firm is
$$NPV = -200 + \frac{100}{1.5} = -133.33.$$
Thus the gamble is value destroying in total.

However, the payoff split is very different. Bondholders receive 300 only in the success state, so their expected payoff is 30, with present value

$$\frac{30}{1.5} = 20.$$

Shareholders receive $1000 - 300 = 700$ in the success state and 0 otherwise, so their expected payoff is 70, with present value

$$\frac{70}{1.5} = 46.67.$$

Therefore, shareholders prefer the negative-NPV gamble because increasing volatility increases the value of their call-like equity claim, while much of the downside is borne by bondholders.

Merger structuring can also be illustrated with contingent value rights. In the General Mills–Pillsbury transaction, the seller received shares together with a contingent payment that covered any shortfall below 42.55 one year later, up to a maximum payment of 4.55. That structure can be interpreted as a long put with strike 42.55 combined with a short put with strike 38. The option language is useful because it reveals immediately that the contingent payment provides downside protection over a limited range rather than full insurance.

The same habit of thought also applies to real investment projects. Traditional NPV analysis often ignores flexibility, whereas option-based thinking highlights timing, expansion, contraction, and abandonment opportunities. Financial-option tools provide the foundation for that broader interpretation.

Lecture 31 — Monday, March 24, 2014

Warrants and Convertible Bonds

Option ideas extend to corporate securities issued by the firm itself. The central themes are dilution, hybrid-security valuation, and the strategic reasons firms choose warrants or convertibles instead of plain debt or plain equity.

Definition 31.1 A **warrant** is a call option issued by the firm itself that gives the holder the right, but not the obligation, to buy newly issued common shares from the company at a fixed price within a stated period.

Warrants usually have longer maturities than exchange-traded call options. They are often attached to privately placed bonds as an equity kicker, attached to new issues of preferred or common stock, or granted to underwriters as part of compensation arrangements. In every case, the economic point is the same: the holder gets upside participation in the firm's equity.

The same variables that increase the value of an ordinary call also increase the value of a warrant, except that dividends reduce warrant value because they lower the stock price without compensating the warrant holder. Thus warrant value rises with the stock price, interest rates, volatility, and time to expiry, but falls with the exercise price and expected dividends.

The crucial difference between a warrant and an exchange-traded call option is that warrant exercise causes the firm to issue new shares. A call option simply transfers an existing share from one investor to another, so it does not alter the number of shares outstanding. A warrant creates dilution because the equity base grows by the exercise proceeds but is then divided into more pieces, as summarized in Figure 22.

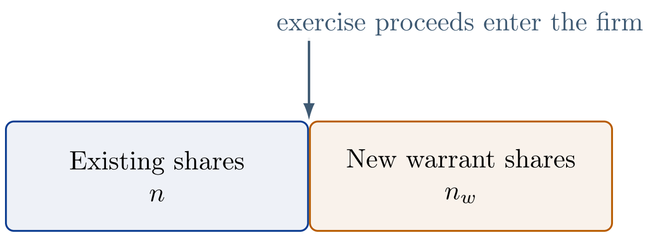

FIGURE 22

Example 31.2 Suppose a firm owns only ten ounces of gold. Two shareholders initially own one share each, and gold has risen so that the firm is worth 3500. Without any warrant exercise, each share is worth

$$\frac{3500}{2} = 1750.$$

Now suppose one warrant allows its holder to buy a new share for 1500, and the warrant finishes in the money. After exercise, the firm has assets worth the original 3500 plus the 1500 cash received on exercise, for total value

$$3500 + 1500 = 5000.$$

However, the firm now has three shares outstanding, so each share is worth

$$\frac{5000}{3} = 1666.67.$$

An otherwise identical call option on an already existing share would be worth $1750 - 1500 =$

250, but the warrant is worth only

$$1666.67 - 1500 = 166.67.$$

Therefore, the warrant is less valuable than the corresponding call because exercise dilutes the value of existing shares.

Because of dilution, a warrant should be worth a bit less than an otherwise identical call option. If n original shares are outstanding and n_w warrants exist, then a common approximation is

$$W = \frac{n}{n + n_w} c,$$

where c is the value of the corresponding call option on an existing share. The factor $n/(n + n_w)$ is the dilution adjustment.

This multiplier can be justified by comparing the gain from exercising a call with the gain from exercising a warrant. The call holder buys an already existing share and captures the full amount by which the option is in the money. The warrant holder buys into a larger equity base after exercise, so only a diluted fraction of that gain belongs to each claim.

Definition 31.3 A **convertible bond** is a bond that gives the holder the right to exchange the bond for a predetermined number of common shares.

A convertible bond is similar to a bond issued together with a warrant, but there is an important institutional difference. A bond with detachable warrants can be split into separate securities, whereas a convertible bond is one inseparable hybrid contract. Its two sources of value are shown in Figure 23; the embedded conversion feature cannot be traded separately from the bond itself.

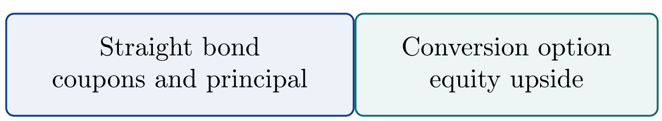

FIGURE 23

Definition 31.4 The **straight-bond value** is the value of the bond if the conversion feature did not exist. The **conversion value** is the value of the shares received upon immediate conversion. The value of the embedded conversion option is the convertible's market value minus its straight-bond value, while the conversion premium is its market value minus its conversion value. The **floor value** is the greater of the straight-bond value and the conversion value.

The market value of a convertible must lie at or above this floor because the bondholder can always choose the more valuable of remaining a bondholder or converting immediately.

Example 31.5 Consider a zero-coupon convertible bond due in ten years with face value 1000, conversion ratio 25, discount rate 10%, current stock price 12, and observed market price 400.

The straight-bond value is
$$\frac{1000}{1.10^{10}} = 385.54.$$
The conversion value is
$$25 \times 12 = 300.$$
Since the straight-bond value is larger, the floor value is 385.54. The option value of the conversion feature is therefore
$$400 - 385.54 = 14.46.$$
Hence the market value decomposes as
$$400 = 385.54 + 14.46.$$

It is tempting to ask whether convertible debt is cheaper than straight debt or straight equity. In an efficient market, that is not the right conclusion. Investors pay for the value of the embedded option, so a convertible should not systematically create financing for less than fair market value.

Still, firms may prefer convertibles or warrants for practical reasons. Convertible debt usually carries a lower coupon rate than otherwise identical straight debt because investors are also buying the conversion feature. Relative to immediate equity issuance, the firm may be able to defer selling equity until later and potentially at a higher stock price. On the other hand, if the stock later performs extremely well, the firm may regret having effectively pre-sold equity through the conversion right.

Convertible issuers are often smaller, higher-growth, more levered firms with weaker bond ratings. The securities are commonly subordinated and unsecured. One plausible interpretation is that convertibles reduce some agency conflicts between bondholders and stockholders because the bondholder can participate in upside if the firm does well.

Example 31.6 Suppose a firm's stock price has fallen sharply, making an ordinary equity issue unusually expensive. Convertible debt can relieve some immediate cash financing pressure while allowing the firm to defer a possible equity issue until investors choose to exercise the conversion feature.

Most convertible bonds are also callable. When the bond is called, investors usually receive a short decision window, often about thirty days, during which they must either convert into stock at the stated conversion ratio or surrender the bond and accept the call price in cash.

Under the shareholder value criterion, the usual forced-conversion trigger is that the conversion value exceeds the call price: bondholders then prefer shares to the cash redemption amount. Firms often wait until the conversion value is substantially above the call price, however, partly because a sharp stock price decline during the decision window could lead bondholders to take the call price instead of converting.

Remark 31.7 The recurring theme is that warrants and convertibles are option-like claims, but they are not identical to exchange-traded options because they affect corporate cash flow, capital structure, and dilution.

Lecture 33 — Friday, March 28, 2014

6. Derivatives and Hedging Risk

Forward and Futures Contracts

Derivative markets have institutional mechanics that connect directly to corporate hedging problems. The main progression runs from simple forward contracts to exchange-traded futures, then to interest rate hedging and swap agreements.

Definition 33.1 A **forward contract** specifies that a commodity or financial asset will be exchanged in the future at a price agreed upon today. It is not an option: both parties are expected to perform.

The buyer is obligated to buy and the seller is obligated to sell at the contracted forward price. Figure 24 emphasizes that the price is fixed today even though the exchange occurs later.

FIGURE 24

If the spot price at maturity is S_T, then the profit to the long side is

$$S_T - F_0,$$

while the profit to the short side is

$$F_0 - S_T.$$

These formulas apply equally well to forwards and futures when we focus on the final settlement value.

Definition 33.2 A **futures contract** serves the same basic economic purpose as a forward contract, but it is standardized and traded on an organized exchange.

Standardization usually covers contract size, quality specifications, and delivery month. Futures also rely on a clearinghouse and daily resettlement, often called marking to market. In Figure 25, the clearinghouse becomes the counterparty to both traders.

FIGURE 25

By becoming the counterparty to both sides of the trade, the clearinghouse greatly reduces default risk: traders post margin and settle gains and losses every day rather than relying directly on one another's credit quality. Initial margin is often only a small fraction of notional contract value, such as about 4% in the examples here, but the clearinghouse can require more if market conditions change.

Positions can be closed in several ways. A trader can reverse the contract by taking the opposite position, can make or take physical delivery, or can settle in cash if the contract is cash settled. In practice, most futures positions are closed out before delivery.

Speculation and Hedging

Futures contracts support two broad uses. A speculator takes a view on price direction. A hedger uses the futures position to offset an existing exposure elsewhere. A trader who expects the price to rise takes a long position, while a trader who expects the price to fall takes a short position.

Definition 33.3 A **long hedge** protects against rising prices, and a **short hedge** protects against falling prices.

For example, a firm that knows it will need to purchase an input later may use a long hedge, while a producer that fears a decline in selling prices may use a short hedge. The offsetting futures positions themselves have the payoff slopes shown in Figure 26.

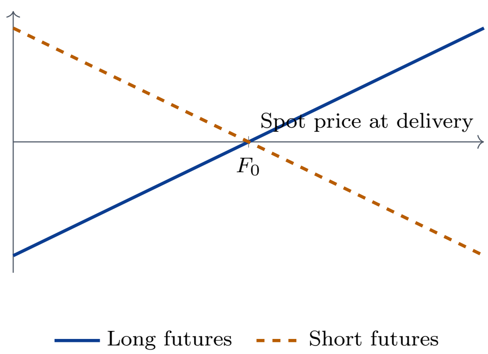

FIGURE 26

Remark 33.4 Hedging is not the same as speculation. The point of a hedge is to reduce the volatility of the overall position, not to create an additional directional bet.

Basis Risk

Definition 33.5 The **basis** is the difference between the futures price and the spot price, defined here as futures price minus spot price.

Spot and futures prices move roughly together and converge at delivery. That convergence, shown in Figure 27, is what makes hedging work.

The residual risk in a hedge is therefore usually basis risk, not outright price risk. By entering the hedge, the firm effectively trades one source of uncertainty for another. Basis tends to be more

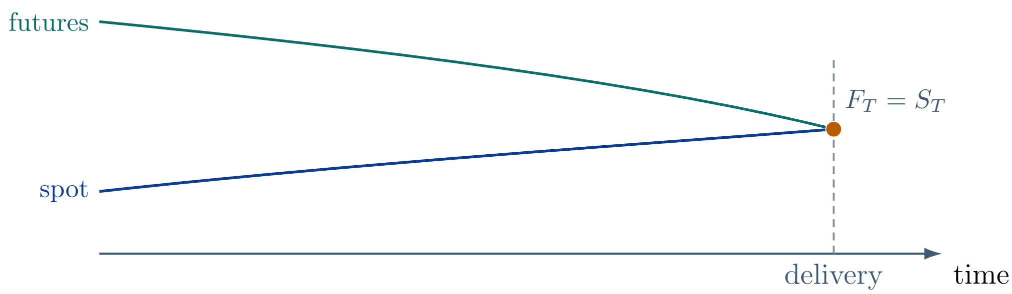

FIGURE 27

predictable than the unhedged spot price, but it is still not perfectly known in advance.

A currency futures example illustrates how daily marking to market works. Suppose the current spot rate is US\$1 = ¥140, the three-month futures price is US\$1 = ¥150, and the contract size is ¥12,500,000. A trader who expects the U.S. dollar to appreciate shorts the fixed yen amount, a position that benefits if the yen depreciates.

If the initial margin requirement is 4%, the original contract value in U.S. dollars is

$$\frac{12{,}500{,}000}{150} = 83{,}333.33,$$

so the initial margin deposit is

$$0.04(83{,}333.33) = 3{,}333.33.$$

Now suppose that one day later the futures rate closes at US\$1 = ¥149. The same yen amount is now worth

$$\frac{12{,}500{,}000}{149} = 83{,}892.62.$$

Relative to the original contracted dollar value of 83,333.33, the position has lost

$$83{,}892.62 - 83{,}333.33 = 559.28.$$

That amount is removed immediately from the margin account, leaving

$$3{,}333.33 - 559.28 = 2{,}774.05.$$

Under the simplified assumption that required margin is reset to 4% of current contract value, the new requirement is

$$0.04(83{,}892.62) = 3{,}355.70.$$

Thus the account is now 581.65 below that simplified requirement. In actual futures markets, initial and maintenance margin levels are set by the exchange or broker rather than mechanically recomputed as a constant percentage each day; a margin call occurs when the account falls below the applicable maintenance level.

Interest Rate Futures

Interest rate futures require a bond pricing foundation. Start with a Government of Canada bond paying semiannual coupon C for T years with maturity value F. Under a flat term structure with annual yield r compounded semiannually, the bond price is

$$P_0 = \sum_{t=1}^{2T} \frac{C}{(1+r/2)^t} + \frac{F}{(1+r/2)^{2T}}.$$

If the term structure is not flat, then each cash flow must instead be discounted at the spot rate for its maturity.

An N-period forward contract on such a bond is priced by carrying the appropriate bond value to the delivery date, after adjusting for any coupons paid before delivery. In words, the forward price equals the current value of what will be delivered at date N, grown at the relevant financing rate. The futures price is usually close to this forward price, although daily resettlement can create a small difference.

The corporate finance motivation is immediate. A mortgage lender that has promised funds in the future at rates fixed today can hedge that exposure by selling the future mortgage cash flows forward or, more practically, by using interest rate futures. The futures market is often cheaper and easier to access than a customized forward transaction.

Another way to hedge interest rate risk is to match the duration of assets and liabilities. Duration measures the effective maturity of the cash flow stream and therefore the sensitivity of value to interest rate changes.

Several properties of Macaulay duration are worth remembering. Longer maturity usually means longer duration, although duration increases at a decreasing rate. Higher coupon rates shorten duration because more cash is received earlier. Higher yields also shorten duration. A zero-coupon bond has Macaulay duration exactly equal to its maturity. In practice, Macaulay duration is interpreted as a present-value-weighted average maturity of the bond's cash flows.

Duration hedging is especially important for institutions such as lenders, pension plans, or insurers that carry large fixed-income positions. The goal is to match the interest rate sensitivity of assets with the interest rate sensitivity of liabilities rather than to hedge each individual cash flow separately.

Swap Contracts

Definition 33.6 A **swap** is an agreement under which two counterparties exchange cash flows at periodic intervals according to a prearranged formula.

Plain-vanilla fixed-for-floating interest rate swaps and currency swaps are the main cases. A swap bank may serve as a broker, simply matching counterparties, or as a dealer, taking one side of the transaction and then laying off the risk later.

Figure 28 isolates the swap cash flows in the example below. The firms' external borrowing remains in place; only the displayed fixed and floating payments are exchanged through the dealer.

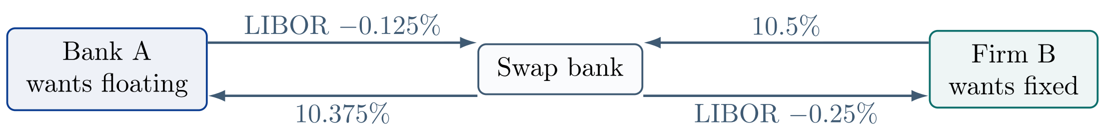

FIGURE 28

Example 33.7 Consider an interest rate swap between two borrowers. Bank A, rated AAA, wants floating-rate funds and can borrow at either 10% fixed or LIBOR floating. Firm B, rated BBB, wants fixed-rate funds and can borrow at either 11.75% fixed or LIBOR +0.5% floating.

The swap bank offers Bank A the following contract on US$10,000,000: Bank A will pay LIBOR −0.125% and receive 10.375%. If Bank A borrows externally at 10% fixed, its net financing cost becomes

$$-10.375\% + 10\% + (\text{LIBOR} - 0.125\%) = \text{LIBOR} - 0.5\%.$$

Thus Bank A saves 0.5% per year relative to borrowing floating directly at LIBOR.

The swap bank offers Firm B the opposite side: Firm B pays 10.5% and receives LIBOR −0.25%. If Firm B borrows externally at LIBOR +0.5%, its net financing cost becomes

$$10.5\% + (\text{LIBOR} + 0.5\%) - (\text{LIBOR} - 0.25\%) = 11.25\%.$$

Thus Firm B also saves 0.5% per year relative to borrowing fixed directly at 11.75%.

The swap bank earns the spread between what it receives and pays on each side, namely

$$10.5\% - 10.375\% = 0.125\%$$

on the fixed leg and

$$(\text{LIBOR} - 0.125\%) - (\text{LIBOR} - 0.25\%) = 0.125\%$$

on the floating leg, for total compensation of 0.25% of notional principal, or US$25,000 per year on each US$10,000,000 notional amount.

Common variations include currency swaps, zero-for-floating swaps, floating-for-floating swaps, and more exotic customized structures. The unifying idea is that a swap can be viewed as a portfolio of forward-like cash flow exchanges across a sequence of dates.

Credit Default Swaps

Definition 33.8 A **credit default swap (CDS)** is a derivative that functions like insurance against a credit event.

The buyer pays periodic premiums to the seller, and if the specified borrower defaults or undergoes another covered credit event, the seller compensates the buyer. The 2012 Greece restructuring illustrates that CDS contracts really do trigger when a formal credit event is recognized.

Derivatives use is widespread in practice even though derivatives often do not appear on the balance sheet in a way that is easy for outsiders to observe. Survey evidence suggests especially heavy use among large public firms, Canadian multinationals, nonregulated firms, and firms in industries such as precious metals, forestry, pipelines, and agriculture. Foreign exchange and interest rate derivatives are the most commonly used types.

Remark 33.9 Swaps can be used to hedge just as futures can. Conceptually, a swap is a strip of forward or futures-like agreements with different settlement dates.

Appendix: Lecture and Tutorial Index